

# **COST RECOVERY FROM ROAD TOLLS WITH LONG-RUN UNCERTAINTY**

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## **Introduction**

A longstanding question in transportation economics is whether the revenues from efficient congestion tolls pay for the costs of transportation infrastructure. The question is of interest from both a positive standpoint (e.g., will a subsidy be required from other sources?) and a normative one (do users in aggregate pay their full costs?).

The seminal result on cost recovery is a theorem due to Mohring and Harwitz (1962). For roads, which are the focus of this paper, the theorem states:

**(Classical cost-recovery theorem)** Toll revenues just suffice to pay for a road of optimal capacity if three assumptions hold: (a) travel costs are homogeneous of degree zero in traffic volume and road capacity<sup>2</sup>; (b) capacity is perfectly divisible and (c) the cost of constructing a road is proportional to its capacity.

Mohring and Harwitz derived the theorem for a deterministic environment. Yet uncertainty is often practically important for roads. Demand and capacity fluctuate unpredictably from day to day due to weather, accidents, unplanned road maintenance and so on. Lindsey (2009) shows that the theorem continues to hold in the face of this uncertainty if two additional assumptions are met: individuals learn

road conditions before deciding whether to travel, and the congestion toll is varied *responsively* to maintain efficient usage levels.

The current paper is concerned not with short-run fluctuations in capacity and demand, but rather with uncertainty about facility costs and the long-run evolution of demand. The costs and time required to build, expand and rehabilitate a road are uncertain. Costs can rise because of changes in technical specifications, new construction methods, demands from municipalities for better network connections and so on (Nijkamp and Ubbels, 1999; Berechman, 2009). Major cost overruns and delays are common for toll roads. In a large international survey, Flyvbjerg et al. (2003) found an average cost escalation of 20.4% for road projects and 33.8% for bridges and tunnels. And in a study of un-tolled roads on Vancouver Island, Wu (2006) determined that 104 of 128 highway projects and 29 of 36 bridge and tunnel projects experienced cost overruns.<sup>3</sup>

Road operations and maintenance costs are also unpredictable. Input costs (labour, fuel and material) can vary significantly over time. Natural disasters cause damage. Climate change affects the frequency and severity of extreme weather, flooding, frost heave and so on. And fluctuations in traffic volumes affect the rate of wear and tear on roads.

Traffic volumes are also a major source of uncertainty. In another large international survey, Flyvbjerg et al. (2006) found that for half of road projects actual traffic deviated from forecasted traffic by more than  $\pm 20\%$ . Traffic volumes are affected by a host of unpredictable factors: project completion time, duration of the ramp-up period, economic growth rates, fuel prices, anticipated land-use developments that may fail to materialize, construction of competing or complementary roads, environmental concerns that curb automobile usage and so on.

Despite improvements in data collection and econometric methods, forecasts have not become more accurate over time (Flyvbjerg et al., 2006; Transportation Research Board, 2006). Optimistic demand projections tend to be the norm for toll road projects. Bain (2009)

identifies several reasons including lower-than-expected travel time savings, over-estimation of drivers' values of time and corresponding willingness to pay tolls, and errors in designing complex tolling schemes in which tolls vary by vehicle type, section of road and time of day.

Technology is a third factor that can affect cost recovery from congestion tolls over the lifetime of a road. Traffic management system techniques such as ramp metering help to regulate demand. Incident Management Systems reduce the duration of traffic incidents. Advanced Traveler Information Systems notify motorists about traffic conditions. As these Intelligent Transportation Systems (ITS) technologies improve, and become more prevalent, congestion is likely to drop. Road vehicles are also becoming smaller, smarter and safer. Vehicle collision avoidance systems, lane-departure warning systems, driver fatigue monitoring systems, heads-up displays, and improved braking systems are reducing the probability of accidents that contribute to congestion. By increasing effective road capacity, and managing demand, all these technologies may result in lower congestion tolls and cost recovery.

A final influence on capacity and cost recovery is flexible road capacity design. The capacity of existing roads can be increased or decreased by re-striping lanes, allowing vehicles to use shoulders during peak periods, changing speed limits, introducing or eliminating features to accommodate public transit and/or bicycles and so on. The appropriate date at which to make these adjustments depends on traffic volumes, ITS technology and vehicle designs, and is therefore unpredictable.

### **The cost-recovery theorem with long-run uncertainty**

In an unpublished paper (de Palma and Lindsey, 2010) we have investigated whether the cost-recovery theorem extends to long-run uncertainty. Our analysis builds on Lindsey (2009) and a seminal study by Arnott and Kraus (1998) who showed that the theorem extends to non-stationary environments if there is no uncertainty. Arnott and Kraus consider a variety of model specifications. The

most general of the specifications for which the theorem extends (and which is reasonably descriptive of roads) is one in which a facility is built from scratch and intermittently expanded as demand grows.<sup>4</sup>

In de Palma and Lindsey (2010) we derive conditions under which the theorem holds in the Arnott and Kraus (1998) model when long-run uncertainty about costs and demand is added. Due to space constraints, we omit the model and derivations, and just state the new theorem with changes and additions from the classical theorem identified in italics:

**(Extended cost-recovery theorem)** *Expected present-value lifetime toll revenues just suffice to pay expected construction costs for a road of optimal capacity if five assumptions hold: (a) travel costs are homogeneous of degree zero in traffic volume and road capacity at any calendar date and in any state; (b) design capacity is perfectly divisible, (c) the cost of constructing a road is proportional to its capacity, (d) realized capacity at any calendar date and in any state is proportional to design capacity, and (e) users have perfect information about the state, and tolls are set responsively to maintain efficient usage levels.*

Although the assumptions of the extended theorem are more stringent than for the classical theorem (as is to be expected) the extended theorem encompasses a wide range of circumstances. It allows for inflation in construction costs and for changes in construction technology as long as constant returns to scale in construction continue to hold. It allows for increases or decreases in demand, changes in the discount rate and changes in the probability distribution of states (e.g., due to a decline in accident frequency). It allows for changes in travel costs due to advances in ITS technology, vehicle design and road design. Finally, the extended theorem encompasses the assumptions of the cost-recovery theorem in Lindsey (2009) so that cost recovery holds in expected values with short-run capacity and demand fluctuations as well as with long-run fluctuations.

In short, the extended theorem is rather robust. However, it establishes only that *expected* present-value toll revenues cover

*expected* construction costs. The theorem says nothing about the degree to which costs will actually be recovered once uncertainty has been realized. Both surpluses and deficits are possible, and the experience with road projects reviewed in the introduction suggests that large departures from cost recovery may be common even if construction costs and demand are forecast without bias. The purpose of the next section, and the main contribution of this paper, is to examine the plausible extent of surpluses and deficits and to identify which parameters affect cost recovery the most.

### **Departures from expected cost recovery**

Departures from cost recovery can occur for two main reasons. One is that construction costs and other parameters can be estimated incorrectly so that roads are built too large or too small. The other is that construction costs and traffic demand are inherently unpredictable so that even an optimally designed road will typically not recover its costs exactly. We consider these two reasons in turn, with emphasis on the first reason.

#### *Bad parameter estimation*

The implications for cost recovery of bad construction cost estimates warrant only brief discussion. The survey by Flyvbjerg et al. (2003) found that actual costs exceeded forecast costs by an average of 20.4% for roads and 33.8% for bridges and tunnels. These figures might suggest that expected cost recovery will run at  $1/1.204 = 0.831$  for roads, and  $1/1.338 = 0.747$  for bridges and tunnels. However, this disregards the possibility that costs were deliberately underestimated (possibly by contractors or political supporters of the projects) in the hope of getting them approved. It also neglects that operations, maintenance and rehabilitation contribute to lifetime project costs. Thus, construction cost overruns alone are unlikely to drive average cost recovery below about 75%.

The effects of errors in other parameters are not as transparent, and to examine them we use a model. For simplicity and clarity, attention is focused on initial construction of a road and operation until the first capacity addition which is assumed to occur at a fixed date,  $T$ .<sup>5</sup> The

interest rate,  $r$ , is assumed to be constant over time. Consistent with assumption (c) of the extended theorem, the cost of building a road with design capacity  $\hat{s}$  is assumed to be  $\gamma\hat{s}$  where  $\gamma$  is a constant. Operations and maintenance costs are ignored, and capacity is assumed not to depreciate.<sup>6</sup>

Demand at time  $t$  in state  $w$  is given by a constant-price-elasticity function  $N_{tw} = n_{tw} p_{tw}^{-\eta}$  where  $n_{tw}$  is demand "intensity",  $p_{tw}$  is the full price or generalized cost of a trip, and  $\eta > 0$  is the price elasticity. The full price is the sum of the user-borne travel cost and the toll,  $\tau_{tw}$ . User cost is given by the widely-used Bureau of Public Roads formula  $C(N_{tw}, s_{tw}) = d_t (N_{tw}/s_{tw})^\chi$  where  $s_{tw}$  is the capacity realized at time  $t$  in state  $w$ .<sup>7</sup> To allow for advances in ITS technology, vehicle design and road design, coefficient  $d_t$  is assumed to decline slowly along an exponential path  $d_t = d_0 e^{-\xi t}$  where parameter  $\xi$  describes the rate of technological progress.<sup>8</sup>

For simplicity, capacity is assumed to be fully available at all times in all states.<sup>9</sup> Demand intensity evolves according to Geometric Brownian Motion, following the stochastic differential equation  $dn_t = gn_t dt + \sigma n_t dW_t$  where  $g$  is the mean growth rate or "drift" parameter,  $\sigma$  is the standard deviation, and  $W_t$  is a Wiener process.<sup>10</sup>

Let  $n_0$  denote demand intensity at time 0 when the road is built. Optimal design capacity works out to

$$(1) \quad \hat{s} = n_0 \left( \frac{\chi}{\gamma} \right)^{\frac{1}{\lambda}} d_0^{\frac{1-\eta}{1+\chi}} (1+\chi)^{-\eta} \left( (e^{hT} - 1)/h \right)^{\frac{1}{\lambda}},$$

where  $h \equiv \lambda(\xi\chi\eta + g + (\lambda-1)\sigma^2/2) - (r + \xi\chi)$  and  $\lambda \equiv (1+\chi)/(1+\chi\eta)$ . Expected cumulative usage,  $U$ , can be written as a function of  $\hat{s}$ :

$$(2) \quad E\{U\} \equiv E\left\{ \int_{t=0}^T N_{tw} dt \right\} = n_0^{\frac{1}{1+\chi\eta}} \left( (1+\chi) d_0 \hat{s}^{-\chi} \right)^{\frac{-\eta}{1+\chi\eta}} (e^{mT} - 1)/m,$$

where  $m \equiv (g + \xi\chi\eta)/(1 + \chi\eta) - \chi\eta\sigma^2 / (2(1 + \chi\eta)^2)$ . Equation (1) reveals that design capacity is proportional to initial demand intensity. This follows from the assumption that road capacity can be built at a constant unit cost, and a property of Geometric Brownian Motion that demand at all future dates is proportional to initial demand. From eqn. (2) the elasticity of expected cumulative usage with respect to design capacity is  $\varepsilon_s^U = \chi\eta / (1 + \chi\eta) \in (0,1)$ . Building a larger road results in greater usage because congestion is reduced. However, usage increases less than proportionally to the increase in capacity except in the limiting case where demand is perfectly elastic (i.e.,  $\eta \rightarrow \infty$ ). Expected present-value toll revenues,  $R$ , can be written as a function of design capacity

$$(3) \quad E\{R\} = n_0^\lambda \chi (1 + \chi)^{-\eta\lambda} d_0^{\frac{1-\eta}{1+\chi\eta}} \hat{s}^{-\chi(1-\eta\lambda)} (e^{hT} - 1) / h.$$

The elasticity of  $E\{R\}$  with respect to design capacity is  $\varepsilon_s^R = -\chi(1 - \eta\lambda)$ . Toll revenue increases or decreases depending on whether demand is elastic or inelastic (i.e.,  $\eta > 1$  or  $\eta < 1$ ). The elasticity of cost recovery,  $\rho$ , with respect to design capacity is  $\varepsilon_s^\rho = \varepsilon_s^R - 1 = -(1 + \chi) / (1 + \chi\eta) < 0$ . Building a larger road reduces cost recovery because the cost of the larger road is not recovered by greater toll revenue.<sup>11</sup> Finally, by substituting eqn. (1) for  $\hat{s}$  into eqn. (3), it is straightforward to show that expected present-value revenues equal capacity costs:  $E\{R\} = \gamma\hat{s}$ .

To gain further insights it is necessary to proceed numerically. We parameterize the model for a three-lane road with a design capacity of  $\hat{s} = 6,000$ . Daily usage in year 0 is assumed to be  $N_0 = 12,000$  so that the peak period lasts for two hours. The generalized cost (net of free-flow cost) in year 0 is  $p_0 = \$7.50$ , and the initial toll is  $\tau_0 = \$5.00$ . Parameter values that support this equilibrium are  $\chi = 2$ ,  $\eta = 0.25$ ,  $d_0 = 0.625$ ,  $n_0 = 19,858$  and  $\gamma = 131.93$ . The time interval before the first capacity addition is set to  $T = 15$ , and the discount rate is set at  $r = 0.05$ .

It is difficult to judge a reasonable value for parameter  $\xi$ . There is also little published information on which to base values of  $g$  and  $\sigma$ . Appropriate values will vary by country, rate of national and regional economic growth, rate of growth in automobile ownership, and other factors. Suitable parameterization will therefore vary from case to case. For exploratory purposes here we will set the rate of technological progress to  $\xi = 0.01$ , the mean annual growth rate of demand to  $g = 0.02$ , and the standard deviation of the growth rate to  $\sigma = 0.05$ . Alternative values of  $\xi$ ,  $g$  and  $\sigma$  are considered in the sensitivity analysis below.

With these base-case parameters, mean traffic volume over the 15-year time horizon is  $E\{U\} = 74.6$  million. Cumulative expected toll revenues are \$288.9 million which match construction costs *as per* the extended theorem.<sup>12</sup>

To assess how design capacity and expected cost recovery depend on parameter estimates, we now alter the parameter values one at a time while assuming that the true parameter values remain equal to the base-case values. To do this we use eqn. (1) to compute design capacity using the modified parameter values, and then use eqn. (2) and eqn. (3) to compute expected cumulative usage and toll revenues using the true parameter values. Given  $\chi = 2$  and  $\eta = 0.25$ , the elasticity formulas given above yield  $\varepsilon_s^U = 1/3$  and  $\varepsilon_s^p = -2$ . Cost recovery is therefore much more sensitive to design capacity than is cumulative usage.

The results of the sensitivity analysis are reported in Table 1. Each quantity is stated as a multiple of the value that would obtain with correctly specified parameters. The first two rows show the effects of misestimating unit construction costs. Estimated values are marked with a  $\sim$  (tilde) to distinguish them from the true values. Thus,  $\tilde{\gamma} = (5/6)\gamma$  indicates that the unit cost is underestimated by one sixth.



Table 1: Sensitivity to parameter errors

Misspecified parameter	Design capacity	Expected cumulative usage	Expected cost recovery rate
(1) Unit construction cost			
$\tilde{\gamma} = (5/6)\gamma$	1.095	1.031	0.833
$\tilde{\gamma} = (7/6)\gamma$	0.926	0.975	1.167
(2) Initial demand			
$\tilde{n}_0 = (2/3)n_0$	0.667	0.874	2.25
$\tilde{n}_0 = (4/3)n_0$	1.333	1.101	0.562
(3) Mean growth rate in demand (true value: $g = 0.02$ )			
$\tilde{g} = 0$	0.873	0.956	1.313
$\tilde{g} = 0.01$	0.933	0.977	1.150
$\tilde{g} = 0.03$	1.076	1.025	0.863
$\tilde{g} = 0.04$	1.163	1.052	0.739
(4) Standard deviation of demand growth (true value: $\sigma = 0.05$ )			
$\tilde{\sigma} = 0$	0.991	0.997	1.018
$\tilde{\sigma} = 0.1$	1.027	1.009	0.947
(5) Demand elasticity (true value: $\eta = 0.25$ )			
$\tilde{\eta} = 0.125$	1.017	1.006	0.966
$\tilde{\eta} = 0.375$	0.984	0.995	1.032
(6) Rate of technological progress (true value: $\xi = 0.01$ )			
$\tilde{\xi} = 0$	1.037	1.012	0.930
$\tilde{\xi} = 0.02$	0.965	0.988	1.073

The first row of Table 1 shows that if construction costs are underestimated by 1/6 (about 17%) the road is overbuilt by 9.5%. Expected usage increases, but only by a little over 3%.<sup>13</sup> The largest effect is on cost recovery which changes in proportion to  $\gamma$ . Overestimating construction costs by 1/6 has a similar but opposite effect on capacity, cumulative usage and cost recovery.

Row (2) of Table 1 shows that errors in estimating initial demand have a pronounced effect on expected cost recovery. If demand is underestimated by 1/3 (which is not uncommon in practice) the road is built to 2/3 of its optimal capacity. Construction costs are reduced commensurately. The toll, meanwhile, rises considerably because of the nonlinear dependence of congestion delay on capacity. The end result is that over the 15-year period the road generates an expected revenue surplus equal to 125% of its cost. By contrast, if demand is overestimated by 1/3 (also not unrealistic), expected toll revenues cover little more than half construction costs.

Row (3) assesses the effects of the mean growth rate in demand. As expected, underestimation of growth results in a smaller road capacity, lower cumulative usage and higher expected cost recovery. If growth is disregarded, the road recovers on average nearly one third more than its costs. By contrast, if growth is overestimated by 2 percentage points, expected cost recovery falls 26% below costs.

Row (4) of Table 1 shows that misestimating variability in the growth of demand has little effect on design capacity, cumulative usage or cost recovery. Similarly, row (5) shows that varying the demand elasticity up or down by 50% has only small effects. The rate of technological progress does not have a major influence either (cf. Row (6)). Misestimating the rate of progress by  $\pm 1$  percentage point has about the same effect as misestimating the mean growth rate of demand by about  $\pm 0.5$  percentage points in the opposite direction.

### **Probability distribution of revenues and cost recovery**

The analysis to this point has focused on deviations from expected cost recovery due to errors in estimating model parameters that determine construction costs, congestion or demand. We now turn briefly to the second question of how much actual (i.e., realized) revenues and cost recovery for an optimally designed road can differ from their expected values due to the inherent variability of demand.

One simple way to address this question is to consider the probability distribution of revenues at a given point in time. Doing so gives only a partial view of how much cumulative revenue is generated over a road's full lifetime. However, it has the advantage that closed-form analytical solutions exist. Table 2 presents summary statistics given the base-case parameter values for aggregates of interest in year 10 (i.e., two thirds of the way through the 15-year time horizon).

Uncertainty in the model is driven by stochastic growth in the intensity of demand,  $n$ . By year 10, expected intensity has increased to  $n_0 e^{gt} = (19,858) e^{(0.02)10} = 24,255$ . The coefficient of variation (CV) of demand intensity at  $t=10$  is about 0.16. The CV for usage is smaller because congestion has a dampening effect on usage. The CV for the toll is larger because usage has a nonlinear effect on congestion delay, and usage and delay vary in tandem. Revenue has the largest CV of all because it is the product of usage and the toll which also vary in tandem. Expected daily revenues in year 10 are \$83,005. This is some 38% higher than the revenues of \$60,000 generated just after the road is built. Nevertheless, because of the large variability in demand there is a 0.19 probability that actual revenues in year 10 are below their starting level.

Table 2: Variability in daily traffic, toll and toll revenue in year 10

Variable	Mean	Standard deviation	Coefficient of variation
Demand intensity	24,255	3,859	0.159
Traffic	14,129	1,489	0.105
Toll level	\$5.75	\$1.23	0.213
Toll revenue	\$83,005	\$26,841	0.323

To derive the probability distribution of cumulative revenues over the full time horizon it is necessary to use numerical methods. This was done by solving the stochastic differential equation for  $n$  using small (monthly) time increments over the 15-year horizon, and taking a random draw for the Wiener increment,  $dW_t$ , at each monthly step.

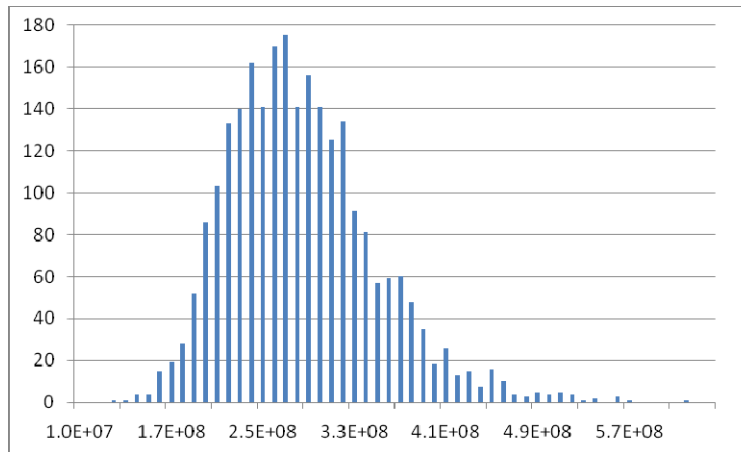
The process was repeated 2,500 times to generate a reasonably smooth probability density function.

The probability distribution function so generated is shown in Figure 1. The coefficient of variation of the distribution is 0.226. This is somewhat smaller than the CV for revenues in year 10 alone, but still appreciable.

### **Conclusions**

In a companion paper (de Palma and Lindsey, 2010) we have shown that expected present-value congestion tolls for an optimally designed road pay for its construction under various types of uncertainty. This paper has focused on the size of potential surpluses or deficits that can arise either from errors in estimating key parameters or from the inherent variability of demand. Using a parametric model we determined that cost recovery is quite sensitive to estimated initial demand, and moderately sensitive to the estimated mean growth rate of demand. The rate of technological progress does not appear to have as strong an effect. Even with no errors the natural variability in demand can result in substantial surpluses or deficits over the lifetime of a road. Further research is clearly warranted using more detailed and accurate information on the causes and magnitudes of the various uncertainties.

Figure 1: Probability density function for cumulative present-value revenues (base-case parameter values)



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## Endnotes

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<sup>2</sup> With homogeneity of degree zero the cost of a trip is unaffected if volume and capacity change by equal proportions.

<sup>3</sup> As reported by Berechman and Chen (2011).

<sup>4</sup> See Section 4.4 of Arnott and Kraus (1998).

<sup>5</sup> These assumptions are consistent with the extended theorem.

<sup>6</sup> To account for the fact that initial capacity remains useful after time  $T$  the unit cost of capacity can be deflated to  $\gamma' = \gamma(1 - e^{-rT})$ .

<sup>7</sup> The constant term in the BPR formula is omitted to facilitate analytical solutions. The user cost function therefore specifies the variable or congestion-dependent component of travel cost while omitting the free-flow component.

<sup>8</sup> Parameter  $\chi$  is included in the exponent of the expression so that  $d_i$  can be interpreted as a multiplicative capacity expansion factor.

<sup>9</sup> As noted above, the extended theorem generalizes the cost-recovery theorem in Lindsey (2009) so that introducing short-run capacity (or demand) fluctuations would not affect the results.

<sup>10</sup> Geometric Brownian Motion is frequently assumed in the options and option value literature (Dixit and Pindyck, 1994) as well as in some transportation infrastructure studies (e.g., Rose, 1998; Saphores and Boarnet, 2006). Marathe and Ryan (2005) find empirical support for Geometric Brownian Motion in the case of usage of two established services (electric power consumption and airline passenger enplanements), but not for two emergent services (cellphones and the Internet).

<sup>11</sup> In the limit as demand becomes perfectly elastic ( $\eta \rightarrow \infty$ ),  $\varepsilon_s^o$  converges to zero. In this limiting case, cost recovery is independent of capacity because demand increases proportionally with capacity, and the toll then does not change.

<sup>12</sup> Total construction costs are  $365 \gamma s = 365 * 131.93 * 6,000 = 288.9$  million.

<sup>13</sup> The relative magnitudes of these changes is approximated by the point elasticity  $\varepsilon_s^U = 1/3$ .