THE IDENTIFICATION OF PARTIAL CAPACITY BRAESS PARADOXES: IMPLICATIONS FOR ROAD DISINVESTMENT

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Introduction

Built primarily in the post-war period during the 1950s and 1960s Canadian roads are nearing their functional lifespan. To complicate matters over the last half-century maintenance funding has not kept up with costs. Canadian municipalities own 60% of total public road infrastructure, but only collect 8% of total government tax revenues (Federation of Canadian Municipalities, 2012). This has led to an estimated $123 billion dollar gap in the spending necessary to properly maintain Canada's transportation networks (Mirza, 2007). At present 1 in 5 municipal roads are listed in poor condition or worse (CCA et al, 2012).

To reduce future budgetary strain, municipal governments throughout Canada have begun assessing options to replace or remove rather than continually maintain transportation assets. Such a notion is new to transportation management as past practice was focused on adding rather than reducing road capacity. This notion is known in planning practice as disinvestment. Perhaps the best example of disinvestment in a Canadian context is highlighted by current city council plans in Toronto, Montreal, and Vancouver to explore the potential impacts of a future without their downtown elevated expressways (Church, 2013; CTV, 2011; Ip, 2013).

The key barrier to moving ahead with disinvestment is the potential for increased traffic congestion. If sustained growth in congestion results from infrastructure retirement it would be economically devastating to a city, as current congestion costs in Toronto are already estimated at $3.3 billion dollars a year in lost productivity (OECD, 2010); and Montreal and Vancouver presently experience even greater levels of traffic congestion than Toronto (Frisk, 2013). Therefore future disinvestment scenarios may rely on a process of weighing the costs of congestion against the costs of continued maintenance. Reduction without disruption is ideal.

In light of this dilemma the aim of the research is to identify opportunities for disinvestment where a capacity reduction will not only limit disruption, but moreover may in fact actually decrease the cost of congestion as a direct result of its loss. To accomplish this counterintuitive task one must identify linkages in the road network that present what is known as a Braess Paradox (Braess, 1968). Braess Paradoxes exist where the presence of excess road capacity leads to increases rather than decreases in overall network travel time.

The outline of the paper is as follows: first a brief review of Braess Paradox and related empirical examples; next an overview of methods including the Network Robustness Index model and the road networks utilized in the study; and lastly a detailed discussion of model results, followed by a brief description of the potential applications of the research and further study.

Literature Review

John Glen Wardrop (1952) found that the behaviours of individual travellers on a road network were best represented by a routing strategy whereby each individual pursues the path that minimizes his or her travel time. This type of non-cooperative behaviour leads to a Nash Equilibrium where an individual vehicle’s travel time only can become worse off if it changes its route. Wardrop’s theory forms the basis for possibly the most commonly utilized traffic assignment algorithm in transportation planning: user equilibrium (Sheffi, 1985).

Due to this non-cooperative routing strategy Braess (1968) found that in certain cases the addition of road capacity to a network would paradoxically lead to an increase in overall network travel time. On the flip side, if that link was removed travel times would decrease[1]. A simple proof of this theory is illustrated in the network diagram from Roughgarden and Tardos (2002) recreated below.

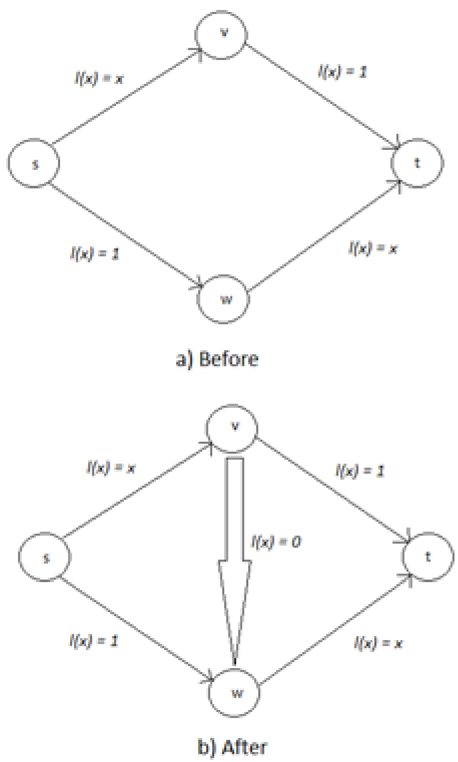

Figure 1: Braess Network, Roughgarden and Tardos, 2002

[1] This is the case of Braess Paradox that we are interested in within the scope of disinvestment. Whether or not the removal of a link may improve network travel times and flow.

In the above network examples (Figure 1) individual vehicles move along the links from origin node s through nodes v and w to destination node t, choosing the routes that produce the shortest travel times. Cost function l represents the travel time on that link, where:

l(x)= x scales linearly by the number of vehicles (i.e. 1 vehicle on the link = 1 unit of time; 2 vehicles = 2 units of time…and so on)
l(x)= 1 represents 1 fixed unit of travel time regardless of the number of vehicles
l(x)= 0 represents 0 fixed units of travel time regardless of the number of vehicles (a "teleporter")

Under the assumption of user equilibrium in example a, one vehicle would traverse the top route and one vehicle would traverse the bottom route from s to t. In this case no user could unilaterally change their route to improve their travel time. This 50/50 split of vehicles on the top and bottom paths results in an identical travel cost per vehicle of 2 units of time from s to t for a system wide cost of 4 units of travel time.

Using the same routing assumption in example b wherein we add a cost free link between nodes v and w or "teleporter" we now find that the system wide travel times increase. In example b the individual choosing to start their journey on the top link will shift routes to follow the zero cost link located at node v. Thus the final leg of the journey to w now costs 2 units of time for each individual, resulting in a total cost of 4 units. Adding in the costs of the first leg of the journey (a unit cost of 1 for each vehicle), system wide costs with the introduction of the new link now become 6 total units, resulting in a Braess Paradox. The difference between the user equilibrium routing in example b versus the optimal routing is represented by the ratio of 6:4 or 3:2, also known as the Price of Anarchy (Koutsoupias and Papadimitriou, 1999).

Although this is a simplified example for descriptive purposes, Braess Paradox holds true for more complex networks, and of course in the absence of zero cost link “teleporters”. Steinberg and Zangwill (1983) proved this the continuity of Braess Paradox on more general networks concluding that “Braess paradox is about as likely to occur as not occur” solidifying Braess Paradox’s existence in theory. This begs the question however; does Braess Paradox exist in reality? And if so (returning to the initial research motivation) can we use it to reduce the amount of infrastructure we must maintain?

Cases of empirically observed Braess Paradoxes are rarely recorded. Nagurney’s review (2005) cites only two. The first recorded case occurred in Stuttgart, Germany, where a new road addition was soon removed after driver complaints of worsening traffic congestion throughout the road network. The second occurred in New York City on Earth Day, 1990, where Times Square was closed off to auto traffic, resulting in improved traffic flow throughout the city [2]. However the identification of Braess Paradox even in these rare cases is complicated by elastic travel demand.

Outside of the scope of Braess Paradox there is an abundance of evidence that link removal can improve overall network flow. Cairns et al (1998) identified over 70 cases where restricting or removing road links from vehicle use has resulted in reduced traffic congestion. In these cases it the author attributes the travel time savings to changes in driver demand rather than Braess Paradoxes. A conclusion that was validated by traffic counts in the surrounding area. So far elastic demand has complicated the discovery of a true Braess Paradox on road real networks. Yang and Bell (1997) proved that even in theory Braess Paradox can only occur within a certain range of demand.

Despite this obstacle, authors have continued search for Braess Paradoxes under the most realistic conditions possible. For example Youn et al (2008) worked with the road networks of Boston, New York and London, and by their estimation several links on each

[2] Times Square has since been partially closed to traffic to create pedestrian walk areas since 2009.

network were likely to result in a Braess Paradox. Rapoport et al (2009) used real driving scenarios in laboratory experiments and concluded that the resulting behaviour would likely create Braess Paradoxes in the real world.

Zhu et al (2010) state that the lack of validated evidence has led some to conclude that, "Braess Paradox is too extreme to be a real world phenomenon." However one may argue that even though empirical evidence case is rare, it may not necessarily mean that Braess Paradoxes do not exist. Instead it may only mean that validation is difficult. Even if a Braess Paradox is identified on a real network through careful study; at this juncture it is unlikely that any transportation authority would be willing to remove such links just to validate results for the betterment of science. However if Braess Paradoxes do occur on real networks the existence of this phenomenon may be part of the key to conquering the maintenance deficit problem outlined in the introduction.

Methods of Analysis

The above literature review highlighted two key problems that must be overcome to move the study of Braess Paradox from the theory to practice. One, Braess Paradoxes only hold if demand remains constant, thus the demand used to identify a Braess Paradox in a model should not change when applied in reality. This is done easily in theory through a simplifying assumption but is much more difficult when applied to real traffic networks. Two, the presence of Braess Paradoxes should be able to be validated by matching empirical observations. Again this is much more difficult in real world networks as complete link removal is uncommon making post removal travel time observations even more unlikely.

To overcome these problems the research below takes the novel approach of identifying Braess Paradox links under partial rather than full reductions in road capacity. To visualize such a partial loss of capacity imagine a scenario where one or two lanes of a multi-lane road were removed. The advantages of identifying Braess Paradoxes under such partial loss conditions are: 1) the limited disruption of a partial rather than full loss of capacity on a link is less likely change aggregate network travel demand, i.e. demand should remain relatively in constant both theory and practice, and 2) planned partial losses in road capacity ostensibly occur all the time as a result of road construction, this allows for much easier validation than complete link removal[3].

To identify instances of Braess Paradox as the result of partial link removal we employ the Network Robustness Index (NRI) Calculator developed by Scott et al (2006). A mathematical definition of the NRI of link a can be expressed as follows:

$$NRI_a = C_a - C$$

where $C_a$is the total system travel time for all vehicles after link a is disrupted and system traffic has been reassigned in the traffic assignment model to a new equilibrium. C is the total system travel time for all vehicles when all links are present and operational in the network (i.e., base-case scenario).

$$C = \sum i \epsilon I \ t_i x_i$$

where t_i is the link travel time for all vehicles on link i, in minutes per trip, and x_i is the traffic flow on link i for all vehicles at user equilibrium. I is the set of all links in the network.

$$C_a = \sum i \epsilon I/a \ \ t_i{}^{(a)} x_i{}^{(a)}$$

where $t_i{}^{(a)}$ is the new travel time across link i when link a has been disrupted, and $x_i{}^{(a)}$ is the new flow on link i. Essentially, NRI is the

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[3] Validation is of the Braess Paradoxes identified within this study are left for further research.

difference in total system travel between the re-route scenario and the base-case scenario.

The above calculations were incorporated into a computer program coded in Caliper script and processed in TransCad, a Geographic Information Software used for transportation planning. TransCAD requires two main inputs to perform traffic assignment: 1) a network and 2) an origin-destination matrix of demand. The use of this application to uncover partial reduction Braess Paradoxes follows below.

The NRI Calculator program was first tested on a theoretical network to detect whether or not Braess Paradox links under partial capacity removal scenarios was possible. This hypothesis testing was conducted mainly due to the computational restrictions of running such scenarios on full networks. The theoretical network itself is based on Christaller's Central Place Theory, where each node represents a city that emanates outwards from a central node. Initially each node was joined to every other linking every node in the network together. Then random links were removed to create a realistic network that is neither too sparse nor too well connected, more accurately reflecting a real world network in theory. Population at each of the nodes diminishes in size as they move further from the central node. These diminishing populations are created by a random number generated within a range. The first order settlement at the center has a range of 600,000 to 550,000 individuals; second order settlements range between 300,000 and 200,000; and third order settlements furthest from the central node have a range between 50,000 and 20,000. Demand between nodes was determined by population size, which was used to create the origin-destination matrix input into the software. Sensitivity analysis was employed the hypothesis test at capacity reductions of 1, 2, 3, 5, 10, 25, 50, 75, and 100 percent levels for 100 iterations of user equilibrium traffic assignment.

Next the NRI Calculator program was run on Toronto's road network to detect the presence of Braess Paradox under partial capacity loss in a real world context. The Toronto network was developed using the 2006 Desktop Mapping Technologies Inc. (DMTI) – CanMap® Route Logistics data and includes the following attributes: travel time, road length, speed limit, number of lanes, and capacity. The demand flows used to create the origin destination matrix is derived from the Transportation Tomorrow Survey (TTS) — a comprehensive survey that includes information about members of the household and their travel in GTHA. The model includes all trips (work/discretionary) originating and ending within the GTHA and covers the morning peak period, starting at 6:00 am and ending at 8:59 am. The speed on the 407 highway was reduced to 80 k/hr to take into account reduced demand due to tolls. Sensitivity analysis across a range of partial reductions was not conducted on this network due to computational weight, instead capacity was reduced at by 25 percent and the program was run at 60 iterations of user equilibrium traffic assignment.

Results

The results of sensitivity analysis on the theoretical network show that Braess Paradoxes do exist when capacity is partially reduced. Moreover that capacity reduction has an impact on the presence of Braess Paradox – as the level of capacity reduction increases the number of Braess Paradox links decrease. Table 1 illustrates the progression of Braess Paradox links and their aggregate effect on system wide travel time as capacity levels are reduced.

Figure 2 indicates that the location of Braess Paradox links change at different levels of capacity reduction. This is consistent with previous findings from Yang and Bell (1999) that Braess Paradox is dependent on changes in demand, and empirical findings from Zhu et al (2010) that a relatively small reduction in capacity on one link can greatly alter system wide travel times. The location of individual Braess Paradox links are highlighted in green for different capacity reduction levels in the figure below. It must be noted that each of these links can be removed in isolation to result in a Braess Paradox reduction of system wide travel time but the result may or may not hold true if more than one of the highlighted links are removed in tandem. This is beyond the scope of the test.

Table 1: Progression of Braess Paradox links at different levels of capacity reduction

Capacity Reduction	Number of Braess Paradoxes	Percent of Braess Paradoxes	Travel Time Reduction (- min)	Total System Wide Travel Time (min)
100	0	0%	0	227156779
75	1	1%	207	220540543
50	4	5%	5309	217407357
25	15	20%	11478	215814470
10	27	36%	24224	215223420
5	30	40%	33255	215075680
3	38	51%	37970	215014438
2	46	61%	50057	214984652
1	50	67%	54246	214960159
0	0	0%	0	215004499

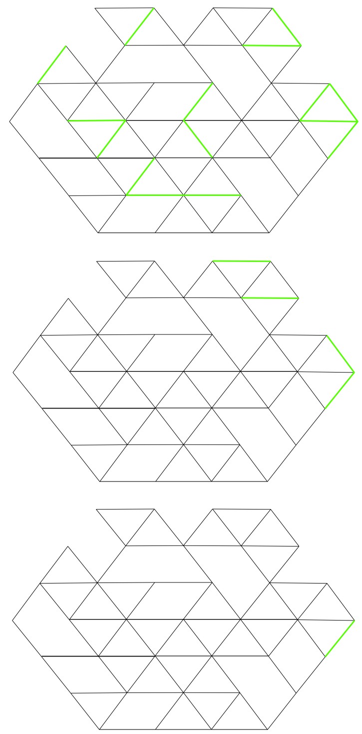

Figure 2: Progression of the location of Braess Paradox links at 25%, 50%. 75% capacity reductions

Figure 3 illustrates the location of Braess Paradox links on the Toronto road network at a capacity reduction of 25%. When capacity reductions are applied to a real network with observed demand inputs the theoretical Braess Paradox results hold true as many such links are identified in the result. The figure below highlights in green areas where Braess Paradoxes occur within the Toronto study area as their capacity experiences a partial reduction. Therefore where each Braess Paradox exists, the Toronto network could be better served by partial removal of at least one of these links in isolation.

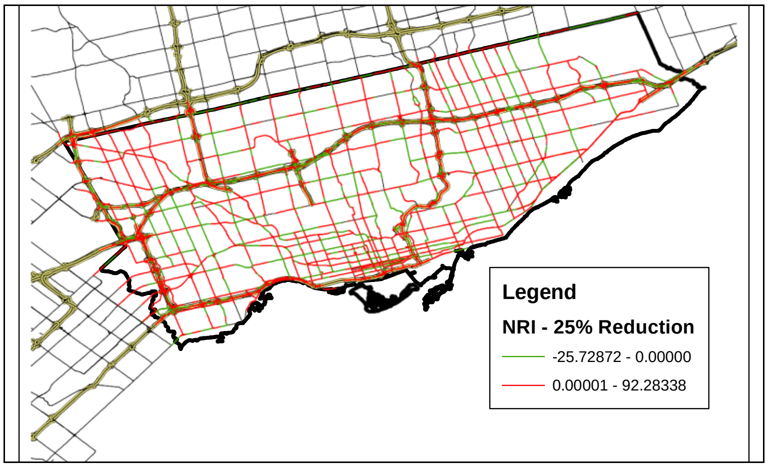

Figure 3: Location of Braess Paradox links on the Toronto road network at a 25% reduction level

Conclusion

Canada has come to potentially a critical point where transportation asset managers must decide on what to do next with roads that are rapidly approaching their functional lifespan. This opportunity may allow for the disinvestment of some roads, thereby reducing the budgetary strain of continual maintenance. Disinvestment however must not exacerbate mounting congestion problems in Canada's growing cities. Thus reduction without disruption is necessary to move forward with this tact. What the above research illustrates is that in some cases it may be possible to do more with less, that Braess Paradoxes may provide the opportunity to reduce capacity and concurrently improve traffic flow.

Further research must be applied to the validation of the above results. Lack of empirical evaluation may be the most significant obstacle to moving forward with disinvesting in identified Braess Paradox links like the ones identified in the results above. However by focusing on partial rather than complete reduction, results may be validated when they coincide with lane closures for construction maintenance.

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