MODELING THE TRUCK TOLL COMPETITION BETWEEN TWO CROSS-BORDER BRIDGES UNDER VARIOUS REGIMES
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Introduction

A new publicly-owned cross-border bridge over the Detroit River between Windsor, Ontario and Detroit, Michigan is planned to be built by the year 2020. The competition between the new bridge (named New International Trade Crossing (NITC), or Detroit River International Crossing (DRIC)) and the existing bridge (the Ambassador Bridge) will have significant impact on international trade and border-crossing traffic between Canada and US. In this paper we model the competition between the two bridges as a duopoly game where each bridge's strategy is its toll level. Due to political constraint, we assume that both bridges will set their passenger car tolls at the highest politically-acceptable level. As such, the passenger car tolls of the two bridges are the same and exogenously given. This assumption is reasonable given that the Ambassador Bridge and the Detroit-Windsor Tunnel charge the same passenger car toll for most of the time. With this assumption, each bridge's strategy reduces to its truck toll level, and the competition equilibrium (i.e. Nash equilibrium) emerges when each bridge cannot improve its objective function by unilaterally changing its truck toll level.

For the Ambassador Bridge, as a privately-owned bridge, its objective function is naturally profit maximization. However, since the new bridge is a publicly-owned bridge it may have different objectives, or, at least profit maximization should not be considered as its only option. In this paper, we consider five different objective functions.
for the new bridge, which will give five different competition regimes between the two bridges, as will be detailed later in the paper.

**Study Area and Data**

Figure 1 shows the transportation network of the study area, which spans over the Windsor-Essex County. The whole study area is divided into 83 traffic analysis zones (TAZ).

Based on official projections, region-wide growth in population and employment has been estimated for the period 2006-2031. The allocation of this growth across the different TAZ was based on the work reported in Gingerich et al. (2014). PM Peak Hour (4 PM) trip productions and attractions for each zone were estimated to create future demand Origin-Destination matrices for passenger vehicles (PV) as well as light (LV), medium (MV) and heavy (HV) commercial vehicles.

Given the emphasis on the border, our analysis focused on the two major players that use the border: private vehicles and heavy trucks. However, for the sake of comprehensiveness and to account for the role of other vehicles on the local road network, light and medium commercial vehicles were also considered. Eventually, solving the toll competition problem for more than two vehicle classes would
prove to be a rather difficult modeling exercise. Therefore, we focused on solving the problem for two vehicle classes while accounting for the role of light and medium commercial vehicles. More specifically, trips pertaining to private vehicles and light commercial vehicles were grouped in one class that we refer to as Cars. On the other hand, medium and heavy commercial trips were also grouped in a second class that we refer to as Trucks. It was convenient to group light commercial vehicles with private vehicles since the former occurred only on local roads and did not cross the Canada-US border. Also, light commercial trips normally take place by small vehicles and therefore like private vehicles can utilize the entire road network. On the other hand, medium and heavy commercial vehicles are constrained to certain roads on the local road network and are normally carried out by large vehicles. Also, since the majority of the non-passenger traffic crossing the Canada-US border pertains to heavy commercial vehicles, grouping medium and heavy trucks was deemed appropriate. The trips for the two modeled vehicle classes were calculated as follows:

\[
\text{Car Trips} = \text{Private Vehicle Trips} + 1.5 \times \text{Light Commercial Trips} \quad (1)
\]
\[
\text{Truck Trips} = \text{Heavy Commercial Trips} + 0.8 \times \text{Medium Commercial Trips} \quad (2)
\]

The 1.5 factor in equation (1) suggests that one light commercial vehicle is equivalent to one and half passenger vehicles. This is an acceptable assumption since light commercial vehicles will normally have more pickup trucks and vans relative to regular size passenger vehicles. On the other hand, the 0.8 factor in equation (2) suggests that one heavy truck is equivalent to 1.25 medium trucks. Following Kanaroglou and Buliung (2008), the passenger car equivalent (PCE) factors for heavy and medium trucks is 2.5 and 2.0, respectively.

**Five Different Competition Regimes**

The competition between the two bridges has a natural bi-level structure, with the upper level being the two bridges setting their respective tolls, and the lower level being the road users (cars and trucks) choosing their routes. This gives rise to an equilibrium problem with equilibrium constraints (EPEC problem). We model the upper level competition equilibrium using the traditional Nash
equilibrium concept, i.e., at equilibrium each bridge cannot improve its objective function by unilaterally changing its strategy (i.e., truck toll). The lower level traffic equilibrium is modeled as a multi-class logit-based stochastic user equilibrium (SUE), where the logit SUE parameter for trucks is set to be sufficiently large so that each truck will choose the shortest path.

As mentioned earlier, we assume the Ambassador Bridge always wants to maximize its profit, while the new bridge may have various objective functions. In the following, we will present five different competition regimes between the two bridges, each corresponding to a different objective function of the new bridge.

To proceed, let us introduce the notations to be used in this paper. In general, we use subscript "P" to denote passenger car, "H" to denote truck (heavy vehicle), "1" to denote the Ambassador Bridge, and "2" to denote the new bridge. A detailed list is given below.

- \( \tau_p \): the common passenger car toll charged by both bridges;
- \( \tau_1 \): the truck toll charged by the Ambassador Bridge;
- \( \tau_2 \): the truck toll charged by the new bridge;
- \( m \): the maintenance cost caused by one truck, assuming equal for both bridges;
- \( \beta_p \): the value of time (VOT) of passenger cars;
- \( \beta_H \): the VOT of trucks;
- \( L \): the set of all links of the network;
- \( v_{ap} \): the passenger car flow on link \( a \in L \); thus, \( v_{1p} \) and \( v_{2p} \) are the passenger car flows on the Ambassador Bridge and the new bridge, respectively;
- \( v_{ah} \): the truck flow on link \( a \in L \); thus, \( v_{1h} \) and \( v_{2h} \) are the truck flows on the Ambassador Bridge and the new bridge, respectively;
- \( t_a = t_a(v_{ap}, v_{ah}) \): the travel time on link \( a \in L \), which depends on the passenger car flow \( v_{ap} \) and the truck flow \( v_{ah} \); thus, \( t_1 = t_1(v_{1p}, v_{1h}) \) and \( t_2 = t_2(v_{2p}, v_{2h}) \) are the travel times...
over the Ambassador Bridge and the new bridge, respectively.

We are now ready to formally present the five competition regimes. Note that the objective function of the Ambassador bridge remain the same under all the five regimes.

**Regime 1:**

\[
\begin{align*}
\text{Ambassador Bridge: } & \max_{\tau_1} \tau_1 v_{1p} + (\tau_1 - m) v_{1H} \\
\text{New Bridge: } & \max_{\tau_2} \tau_2 v_{2p} + (\tau_2 - m) v_{2H}
\end{align*}
\]

subject to

\[(v_{1P}, v_{1H}, v_{2P}, v_{2H})\]

is part of the SUE solution under \((\tau_1, \tau_2)\).

From objective function (3), we can see that under Regime 1 each bridge maximizes its own profit by choosing its truck toll level. The profit of each bridge is equal to the sum of the passenger car toll revenue and truck toll revenue subtracted by the maintenance cost caused by trucks. Note that because the pavement damage caused by passenger cars is negligible compared to that caused by trucks, we ignore the maintenance cost caused by passenger cars.

**Regime 2:**

\[
\begin{align*}
\text{Ambassador Bridge: } & \max_{\tau_1} \tau_1 v_{1p} + (\tau_1 - m) v_{1H} \\
\text{New Bridge: } & \min_{t_2} t_2 (v_{1P}, v_{1H}) v_{1H} + t_2 (v_{2P}, v_{2H}) v_{2H}
\end{align*}
\]

subject to

\[(v_{1P}, v_{1H}, v_{2P}, v_{2H})\]

is part of the SUE solution under \((\tau_1, \tau_2)\).

From objective function (4), we can see that under Regime 2 the new bridge seeks to minimize the total travel time of trucks over the two bridges. In other words, under this regime, we assume the new bridge operator would like to minimize the total border-crossing delay for trucks, i.e., to promote international trade.
Regime 3:
\[
\begin{align*}
\text{Ambassador Bridge: } & \quad \max_{\tau_i} \tau_i v_{1p} + (\tau_i - m) v_{1H} \\
\text{New Bridge: } & \quad \min_{\tau_j} (t_1 v_{1H} + t_2 v_{2H}) \beta_H + (t_1 v_{1p} + t_2 v_{2p}) \beta_P.
\end{align*}
\]
subject to
\[
(v_{1p}, v_{1H}, v_{2p}, v_{2H}) \text{ is part of the SUE solution under } (\tau_p, \tau_1, \tau_2).
\]

In objective function (5), \( t_1 = t_1(v_{1p}, v_{1H}) \) and \( t_2 = t_2(v_{2p}, v_{2H}) \) are written as \( t_1 \) and \( t_2 \) just for simplicity. From objective function (5), we can see that under Regime 3 the new bridge seeks to minimize the total monetary cost of the travel time of both trucks and passenger cars over the two bridges. In other words, under this regime, we assume the new bridge operator would like to minimize the total border-crossing delay for all traffic (including both trucks and passengers). Note that passenger cars and commercial trucks typically value travel time very differently, and thus it is necessary to convert their travel times into monetary values using different VOT parameters, as shown in objective function (5).

Regime 4:
\[
\begin{align*}
\text{Ambassador Bridge: } & \quad \max_{\tau_i} \tau_i v_{1p} + (\tau_i - m) v_{1H} \\
\text{New Bridge: } & \quad \min_{\tau_j} \sum_{a \in L} t_a (v_{ap}, v_{ah}) v_{ah}
\end{align*}
\]
subject to
\[
(v_{ap}, v_{ah}), \ a \in L \text{ is the SUE solution under } (\tau_p, \tau_1, \tau_2).
\]

From objective function (6), we can see that under Regime 4 the new bridge seeks to minimize the network-wide total travel time of trucks.

Regime 5:
\[
\begin{align*}
\text{Ambassador Bridge: } & \quad \max_{\tau_i} \tau_i v_{1p} + (\tau_i - m) v_{1H} \\
\text{New Bridge: } & \quad \min_{\tau_j} \sum_{a \in L} t_a (v_{ap}, v_{ah}) (v_{ah} \beta_H + v_{ap} \beta_P)
\end{align*}
\]

Type: Regular
subject to
\[(v_{ap}, v_{ah}), a \in L\] is the SUE solution under \((\tau_p, \tau_1, \tau_2)\).

From objective function (7), we can see that under Regime 5 the new bridge seeks to minimize the network-wide total monetary cost of the travel time of both trucks and passenger cars.

In summary, comparing the five regimes, we can see that, the new bridge would like to maximize its own profit under Regime 1, minimize the border-crossing delay under Regime 2 and 3, and minimize the network-wide travel delay under Regime 4 and 5. Also, the new bridge cares about trucks only under Regime 2 and 4, and cares about both trucks and passenger cars under Regime 3 and 5. The truck-only setup under Regime 2 and 4 represents the case that the new bridge operator's first priority is to promote international trade between Canada and US.

**Exogenous model parameter value selection**

From the aforementioned five competition regimes, we can see that there are four main exogenous model parameters whose values need to be estimated, the passenger car toll \(\tau_p\), the per-truck maintenance cost \(m\), and the VOT of passenger cars and trucks, \(\beta_p\) and \(\beta_t\). In this study sensitivity analyses for Regime 1 were performed for these model parameters, and the most appropriate parameter values were used when comparing the five regimes.

For brevity the sensitivity analyses cannot be included, and here we just present the selected most appropriate parameter values with brief explanations. Based on historical trend a passenger car toll \(\tau_p = 7.5\) was selected. Following the discussion on truck pavement cost in Holguín-Veras and Cetin (2009), maintenance cost \(m = 1.0\) per truck was selected based on a medium pavement cost value of \$0.72 per mile per truck and the total length of the Ambassador Bridge 1.42 miles (assuming the new bridge will have a similar total length). Based on the calibrated logit model coefficients of travel time and travel cost given by Wilbur Smith (2010), the VOT parameters were
selected as $\beta_p = 0.17$ per minute (or $10.2$ per hour) for passenger cars and $\beta_H = 1.19$ per minute (or $71.4$ per hour) for trucks.

**Solution technique: the Mesh method**

As mentioned earlier, the bi-level competition problem in this paper belongs to EPEC problems, a class of problems well-known to be difficult to solve. Fortunately, in this study, the size of the Windsor-Essex network is small enough for us to use the Mesh method. In the Mesh method, we solve the multi-class SUE traffic assignment under a range of truck tolls for both bridges. Specifically, we change the truck toll of each bridge from 1 minute to 100 minutes at an incremental step-size of 1 minute. This is equivalent to monetary value from $1.19$ to $119$ at a step-size of $1.19$, given the truck VOT parameter $\beta_H = 1.19$ per minute.

We simulate the traffic assignment and obtain the traffic flows under every truck toll pair of the two bridges within this range, totaling 10,000 (100 by 100) pair of tolls. Then, for each given value of $\tau_2$ between $1.19$ to $119$, we can find the $\tau_1$ value $\beta_H = 1.19$ that gives the maximum profit for the Ambassador Bridge. Note that this $\tau_1$ value is the best response strategy of the Ambassador Bridge given the new bridge's strategy $\tau_2$. This way, we can obtain the best response curve of the Ambassador Bridge, $\tau_1$ as a function of $\tau_2$, denoted as $\tau_1(\tau_2)$, when $\tau_2$ changes from $1.193$ to $119.3$. Similarly, we can obtain the best response curve of the new bridge, $\tau_2(\tau_1)$, when $\tau_1$ changes from $1.193$ to $119.3$, for each of the five regimes introduced in last section. Finally, under each competition regime, the intersection of the two best response curves $\tau_1(\tau_2)$ and $\tau_2(\tau_1)$ will be the equilibrium truck tolls of the duopoly competition between the two bridges.
Results and discussion

Using the Mesh method and the selected model parameter values, the competition equilibrium of each regime was numerically identified.

Table 1. Equilibrium under five different competition regimes

<table>
<thead>
<tr>
<th>Regime</th>
<th>Equilibrium Truck Toll ($)</th>
<th>Total Revenue ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ambassador</td>
<td>DRIC</td>
</tr>
<tr>
<td>1</td>
<td>19.09</td>
<td>22.67</td>
</tr>
<tr>
<td>2</td>
<td>17.90</td>
<td>19.09</td>
</tr>
<tr>
<td>3</td>
<td>17.90</td>
<td>19.09</td>
</tr>
<tr>
<td>4</td>
<td>15.51</td>
<td>11.93</td>
</tr>
<tr>
<td>5</td>
<td>15.51</td>
<td>11.93</td>
</tr>
</tbody>
</table>

Table 1 summarizes the equilibrium results of the five competition regimes, from which we can see that the equilibrium results of Regime 2 and Regime 3 are exactly the same, while Regime 4 and Regime 5 also generate identical results with each other. This observation is not a coincidence, but resulted from the disparity in the VOT of the passenger vehicles and the heavy commercial vehicles. That is, because the passenger car VOT is much smaller than the truck VOT, the equivalent change in monetary cost brought by one minute change of the passenger vehicle travel time is much smaller.
than that brought by one minute change of the truck travel time. As a result, taking passenger car travel time into consideration in Regime 3 and 5 will not have much impact on the optimal strategy of the new bridge as compared to Regime 2 and 4.

To see the above explanation more clearly, let us take consider one iteration of the Mesh method to compare the best responses of the new bridge under Regime 2 and under Regime 3. Consider the Ambassador Bridge’s truck toll is fixed at $r_1 = 17.90$, then Figure 2 shows how the objective functions of Regime 2 and Regime 3 of the new bridge change when its truck toll changes. From Figure 2, we can see that the shapes of the two objective functions are identical, both attaining optimality (minimum objective function value) at $r_2 = 16$ minutes (or $19.10).$ Thus it is verified that, whether the new bridge considers trucks only (under Regime 2) or considers both trucks and passenger cars (under Regime 3), its optimal strategy is the same, i.e., it should set a truck toll $r_2 = 19.10$ in both cases given the Ambassador Bridge’s truck toll $r_1 = 17.90$. Similarly, it can be verified that the best responses of the new bridge under Regime 4 and under Regime 5 are the same.

![Figure 2](image-url)

**Figure 2.** Objective functions of the new bridge under Regime 2 and Regime 3 given $r_1 = 17.90$. 

Type: Regular
In summary, because the truck VOT is much higher than the passenger car VOT, whether the new bridge considers minimizing truck travel time only, or it considers minimizing the monetary delay of both trucks and passenger cars, its decision making should not change. This means the five regimes actually reduce to three regimes only. Therefore, from now on, when comparing different regimes, we only refer to Regime 1, Regime 3 and Regime 5.

Figure 3. Best response curves for Regime 1, 3 and 5

Figure 3 presents the best response curve of the Ambassador Bridge, $r_1(t_2)$ (marked as 1(2) in Figure 3), and three different best response curves of the new bridge, $r_2(t_1)$ (marked as 2(1) in Figure 3) under
Regime 1, 3 and 5. Note that the intersection between \( \tau_1(\tau_2) \) and each \( \tau_3(\tau_1) \) represents the truck toll equilibrium under each regime. By comparing the best response curves of the new bridge under Regime 3 and under Regime 5, it can be seen that these two curves are parallel to each other, and both of them intersect with the Ambassador Bridge’s curve at a lower point than Regime 1. This indicates that when the operator of the new bridge focuses on the improvement of border-crossing network efficiency, the toll competition will lead to lower truck toll prices on both bridges.

A few general observations/conclusions can be drawn from Table 1 and Figure 3. First of all, in view of the fact that the current average heavy commercial vehicle toll on the Ambassador Bridge is $26.25, it is clear that with the new bridge, trucks will pay a much lower toll to cross the border due to competition. This is true even if the new bridge only cares about its own profit. Secondly, the more the new bridge cares about the system efficiency, the lower it will set its own truck toll and thereby make both bridges' tolls lower as a result of competition.

In the following we will provide more detailed comparisons among different regimes base on our numerical results.

Figure 4. Revenue comparison under different regimes
Figure 4 illustrates the comparison of revenues under the three regimes. Generally, both bridges experience a decrease in profit from Regime 1 to Regime 5. For Regime 3 and Regime 5, since the objective of the new bridge is not for profit, the operator of the new bridge could choose sacrificing the advantage in revenue to improve the travel time in the study area. However, since the objective of the Ambassador Bridge is profit maximizing for all of these regimes, the most beneficial competition mode is supposed to be the one with highest total revenue. According to Figure 4, it can be seen that the Ambassador Bridge’s profit peak appears in Regime 1. This indicates that after the new bridge is completed, the highest equilibrium revenue of Ambassador Bridge will only occur when the new bridge also goes for profit in the competition.

Figure 5. Truck toll comparison under different regimes

Figure 5 shows the comparison of the equilibrium truck tolls of these three regimes. It can be seen that, from Regime 1 to Regime 5, there is a more significant decreasing trend of the toll price on DRIC as compare to the Ambassador Bridge. This is because the design capacity of the DRIC Bridge is considerably larger than the Ambassador Bridge, and the system congestion level can be decreased if more trucks are attracted to DRIC by its lower tolls.

Figure 6 provides the comparison of the passenger car flows among different regimes. As can be seen from the figure, the Ambassador Bridge attracts the highest passenger vehicle volume in Regime 5,
when the operator of the DRIC Bridge plans to minimize the entire traffic system’s travel time. On the other hand, the peak of passenger vehicle volume of the DRIC Bridge appears in the first regime, when both bridges compete for profit.

![Passenger car flow comparison under different regimes](image1.png)

**Figure 6.** Passenger car flow comparison under different regimes

![Truck flow comparison under different regimes](image2.png)

**Figure 7.** Truck flow comparison under different regimes

Figure 7 gives the comparison of the truck flows among different regimes. It can be seen that the truck volume of the Ambassador Bridge experiences a dramatic decrease from Regime 1 to Regime 5. On the contrary, in the last regime, the truck trip rate on the DRIC Bridge reaches the highest point. This indicates that when most trucks
are assigned to the DRIC Bridge during the competition, the transportation system efficiency of the border crossing area is the highest.

**Conclusion**

We end this paper by summarizing the major findings. First, whether the new bridge operator cares about international trade only (trucks only) or cares about both international trade and local traffic (both trucks and passenger cars), it will behave the same. The reason is that the truck VOT is much higher than the passenger car VOT, which makes the passenger car travel time not important from a system economic efficiency perspective. Second, with the new bridge, trucks will pay a much lower toll to cross the border due to competition. This is true even if the new bridge aims at maximizing its own profit only. Thirdly, the more the new bridge would like to improve the system efficiency, the lower it will set its own truck toll and thereby makes both bridges' tolls lower as a result of competition. The reason is that the new bridge is designed with a very large capacity, which means a higher utilization of the new bridge will improve the system efficiency.

**References**


