GAME THEORETIC APPROACHES TO URBAN TRAFFIC PLANNING

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Introduction

Urban traffic congestion has become a significant and problematic symptom of city life. The mismanagement of city road networks can lead to significant inefficiencies in terms of wasted time by commuters, as well as unnecessary pollution being emitted from vehicles idling in traffic. In the US, the annual cost associated with traffic delays has been estimated to be $48 billion (Arnott, 1994). Any chance of reducing these delays has the potential to free-up huge quantities of capital for more productive uses. As global populations continue to urbanize, and automobile use continues to rise in many nations, the importance of minimizing the social costs of commuting will only become more important.

Urban traffic congestion has historically been viewed as having solutions in the realm of municipal zoning and regional development strategies. Increasingly, in the modern literature, this problem is being analyzed in economic and mathematical terms. This paper will explore the existing theories on traffic "games" and go on to suggest general applications for this set of theories. Many results of routing theory are complex and counterintuitive. A nuanced understanding of individual incentives and game theory is vital to designing traffic networks that can best serve the regional population.

When traffic congestion is modeled as many commuters or "players" each trying to independently minimize their total travel time, the methods of game theory can be readily applied to find equilibriums in the system. When these equilibriums differ significantly from what would be considered a socially optimal outcome, there can be space for a central authority to increase the efficiency of the overall system by several possible means.

A 1952 paper by John Glen Wardrop titled, “Some Theoretical Aspects of Road Traffic Research,” began introducing a theoretical framework to network routing games. To this point traffic congestion problems were considered to be a fairly inescapable result of growing urban populations. Wardrop considered that, “It is not always appreciated that in a severely practical subject such as traffic engineering there is need for theory” but that such theory is essential to “progress in any field.” Wardrop’s paper has been widely credited with kick-starting a wide range of exploration into game theoretic approaches to modeling traffic patters.

Wardrop’s paper was followed by a, now widely cited, paper by Charnes in 1957 which looked to bring together Wardrop’s basic principles of traffic networks with the field of game theory. The major contribution of Charnes was proposing the additional assumption that all drivers in a network are acting selfishly to minimize their own costs, and that this may lead to suboptimal outcomes. The idea is that each driver is imparting an external cost onto all other drivers by adding congestion to the roadway, and slowing down overall flow.

Modern research has gone much further in fully applying game theory to networks. Most recently traffic planning theory has had a surprising synergy with the field of computer science. Local computer networks, as well as peer-to-peer file transfer networks can suffer from the same problems as urban road networks as individual computers attempt to minimize the time it takes to transfer data without consideration of how their route choice might influence the efficiency of the overall system. This theoretical equivalency, coupled with the development of computer modeling for regional traffic networks has spurred a rapid expansion of the theory of traffic network planning over the past decade (Awerbuch, 2005; Koutsoupias, Papadimitriou, 1999).

The actual instruments of government intervention that would be necessary to reach optimal traffic pattern outcomes have received much less attention than the mathematical theory. This paper will go on to suggest how a system could –in practice– be optimized with road tolls and road closures by a central planning authority.

Modeling An Unregulated Game

The complexity of a traffic routing game can vary immensely with the details of the network being modeled; however, there is a framework of underlying game theoretic assumption that can be universally applied to this type of competitive game. The assumption that underpins much of the theory of network routing is that when players are free to make their own decisions they will pick the route that minimizes their personal travel times, irrespective of how their choice effects the system as a whole.

Consider a group of drivers who must all travel from point A to point B, but have a choice of different possible routes. The first situation is of an unregulated road system in which any driver is free to take any road they wish for free. In this case the solution will be obvious. A Nash Equilibrium will be reached when no driver can unilaterally change his or her route to reduce their own travel time; in short, the travel time of taking any possible route must be equal. Wardrop characterizes the solution as follows, "The journey times on all the routes actually used are equal, and less than those which would be experienced by a single vehicle on any unused road (1952)." The player's strategies are characterized by each player choosing the route that will minimize their personal expected travel time. This is akin to check-out lineups at a grocery store. An individual shopper will always pick the shortest line available to minimize their wait time, resulting in all lines being of approximately equal length. It is important to recognize that although this state of affairs may be optimal from a personal perspective it is unlikely to be optimal from a social planning perspective.

Understanding why this outcome is suboptimal involves invoking game theory. Classical economics –within the boundaries articulated

by say Adam Smith in The Wealth of Nations (1776)– asserts that if each individual acts in their own self-interest that an efficient outcome will result due to the so-called “invisible hand” of the market. This notion applies poorly to a market which involves externalities, public goods, and strategic interaction, such a market can be better described by game theory.
Consider the following road network as an illustration:

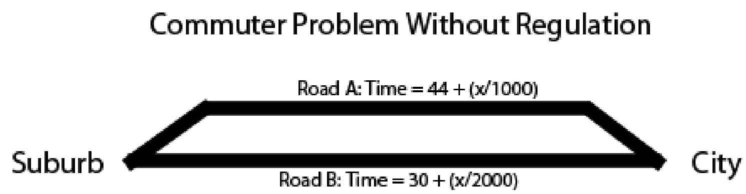

Figure 1: This hypothetical road network assumes that all drivers begin their trip in the “Suburb” and end their trip in the “City.” Travel times of either road are represented by equations which take into account the number of other cars (x) using that particular road.

Suppose that 58,000 commuters must make the trip each morning from the “Suburb” to the “City” (figure 1). All commuters are free to make the choice of whether to take Road A or Road B. The travel time for both roads is a function of distance, road capacity, and the number of other cars on that road (x). The system will reach a Nash Equilibrium when none of the 58,000 drivers can decrease their drive time by switching roads. This system is solved in Appendix A. The solution is found to be 54 minutes of travel time for every driver.

Social Optimum

The previous section examined the normal state of affairs for road networks; open access, where every driver is free to minimize costs without consideration for the other drivers in the system. Youn et al. point out that, “[open access] does not mean that flows in transportation networks minimize the cost for all users as is sometimes assumed … the flows can in reality be far from optimal” (2008). This outcome may not be optimal in that it fails to minimize total drive time for society. This section will demonstrate how the system could become more efficient if a central planner were given the ability to pick driver's routes for them.

Consider the example given in the previous section (figure 1). This simple system could instead be solved by minimizing with respect to the combined number of minutes traveled by all the commuters. This calculation is executed in Appendix B. The optimal solution is shown to be the situation in which 14,667 drivers use Road A and 43,333 drivers use Road B, with an average drive time of 53 minutes 26 seconds. Notice that this is both unique from the Nash Equilibrium outcome as well as lowers the average driving time.

The benefits that accrue due to this centralized planning can best be expressed in the amount of time saved by society. The average driver saves 34 seconds per commute, and there are 58,000 commuters. The cost savings is therefore (34 seconds • 58,000) 548 hours saved for every commute.

Arnott analyzed real traffic networks in American metropolises to make similar rough estimates of the total savings that could be obtained due to a centrally planned traffic network (1994). Arnott goes on to invoke studies which have shown that drivers have an average willingness to pay of about $13 per hour (figure updated for inflation) to avoid an additional hour of driving. Applied to this paper's specific example this gives $7,100 in benefits for every commute. Arnott suggests a figure of 6 billion hours in potential time savings per year across the US, or about 78 billion USD in annual savings. Calculating the potential benefits from instituting centrally planned traffic routing is within the realm of normal cost-benefit analysis. Additionally, the potential for societal cost savings is seemingly quite large, this should act as motivation for the pursuit of such policies. The question of how government can effectively control traffic flows is a separate question that will be addressed later in this paper.

Braess's Paradox

One of the more fascinating and counterintuitive results of traffic routing analysis is known as Braess's Paradox. By analyzing theoretical routing situations Dietrich Braess found that situations exist in which adding an additional road to a network can make every individual driver worse off (Braess, 1968). This is a stronger statement than saying a new road could lower the efficiency of the overall system.

To see how this could come about we turn to an example:

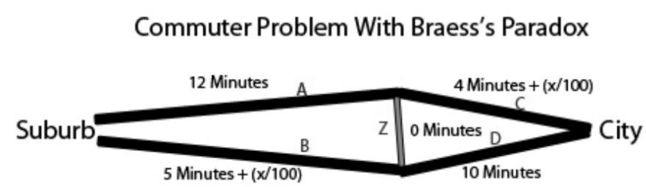

Figure 2: This road network is divided into 5 segments, each with an individual cost function. Some functions are flat times (A, Z, and D) and some are functions of the number of other cars on the road (B and C).

Suppose 500 drivers must travel from a suburb into the city each morning for work. The road network is modeled in figure 2, with the travel times written beside each road. Two of the roads (A and D) do not suffer from traffic congestion; perhaps they have a sufficiently large number of lanes so they never reach capacity. Roads B and C however, do suffer from traffic congestion and therefore have their travel times written as a function of 'x,' which will stand for the number of commuters who take this particular road on a given morning. For the time being ignore the connector road Z.

If drivers are left to choose whichever road they please, and we assume that each driver will take the route that will minimize his or her travel time, then the system will reach a Nash Equilibrium when the travel times of both routes are equal. This system is solved in Appendix C, and it is shown that every driver will take 18 minutes to reach the city, in a Nash Equilibrium.

Now suppose the local traffic authority opens up road Z to traffic. The diagram is not to scale so imagine connector road Z is very short and takes a negligible amount of time to drive. Intuitively it would seem that adding additional capacity to the system should allow commuters to reorganize to lower their travel times; however, it can be shown that the addition of this roadway actually makes every single driver worse off than before.

Allowing each driver to cost minimize again, and reach a Nash Equilibrium will yield a new solution. The solution is more intuitive now. Each driver faces two distinct choices: take either road A or B for the first leg of the journey, then take either road C or D for the second leg. Notice that the solution is simple, and that even if every driver takes road B it is still faster than road A and, likewise, road C is always faster than road D. As a result, all 500 drivers will take the route B-Z-C, and this will take 19 minutes. This is a new Nash Equilibrium because if any driver switches from road B to A, they will add time to their trip, and likewise if they chose to switch from road C to D. No driver has an incentive to switch and the system has therefore settled into a Nash Equilibrium. This demonstrates that adding Road Z is harmful to the efficiency of the system.

After Braess suggested the existence of this situation there have been many studies attempting to uncover whether this "paradox" is a practical problem for traffic planners. Potentially, the complications of a real life traffic network make such a situation unlikely to occur, in which case Braess's Paradox would be more of an intellectual curiosity (Arnott, 1994; Rapoport, 2005; Youn, 2008).

Rapoport et al. ran laboratory experiments attempting to test the descriptive power of Braess's Paradox. Subjects chose routes in subsequent rounds of a game, unaware of the true cost functions of each route. The true makeup of the game was similar to what has been modeled in figure 2 of this paper. When there are two separate routes ("upper" or "lower") Rapoport found the expected equilibrium occurring, in which the travel times for the two roads are equal. When a new road was added to invoke Braess's Paradox (akin to road Z in this paper) subjects shifted to the new Nash Equilibrium where all subjects take the same route, and the cost to each player increases. These results are consistent with Braess's Paradox, and the result approached this equilibrium within only a few rounds of play.

Following this experiment Rapoport introduced a more complex road system, which required players to make several choices between roads. This new network theoretically still had the characteristics for Braess's Paradox to emerge. What he found was that the subjects did shift gradually towards the new (higher cost) equilibrium, but even after 80 rounds of play the new equilibrium was never fully realized. The participants were not playing rational best responses. This suggests that if the road network is quite complicated (as most real world networks are) then drivers may be unable to fully realize their best responses. This means there are now stochastic and irrational elements to play. Without the assumption of rationality the application of game theory in general may be suspect. The extent to which the Braess's Paradox prediction was violated was not quantified in Rapoport's paper, but it seems that a strict application of game theory can still offer a good approximation for most traffic situations, and offer very accurate solutions in the case of simple networks.

General Network Analysis and The Price of Anarchy

In analyzing possible efficiency gains in road networks it is useful to consider how much more efficiently the network could run if we were to compel the drivers to take up the socially optimal solution. The disparity that exists between the socially optimal outcome of a road system and the Nash Equilibrium outcome has been defined in the literature as the Price of Anarchy. This notion was first articulated by Koutsoupias and Papadimitriou in 1999, although they did not call it "Price of Anarchy." Awerbuch et al. introduce this theory in a paper, and describes it by stating:

"The degradation of network performance caused by the lack of a centralized authority can be measured using the worst-case coordination ratio (price of anarchy) … which is the ratio between the worst possible Nash Equilibrium and the social optimum." (Awerbuch et al., 2005)

An example of an explicit calculation of the Price of Anarchy can be found in Appendix D. Interestingly Awerbuch is not addressing issues of urban traffic flow in this paper but is discussing routing of computer networks transferring data. Much of the modern literature regarding the Price of Anarchy in routing games is centered on attempting to optimize routing of computer data. Awerbuch, as well as others, have explicitly pointed out that this problem can be modeled identically to urban traffic flow. This is because –much like drivers– computers in a network search for the quickest route, irrespective of the potential externalities this places on the network as a whole. As a result of these similarities these two, seemingly disparate fields of study, make use of much of the same literature.

The Price of Anarchy is an interesting statistic when applied to traffic planning because it can be used to make a case for or against intervention by government into the incentives of drivers. If the Price of Anarchy is very high, market intervention could potentially allow for large efficiency gains. If rival government interventions are being considered for several different areas then intervention in the section which possesses the largest Price of Anarchy should be instituted with more urgency.

Optimal Tax Policy

Now that it is clear there can be a difference between the socially optimal network outcome and the outcome that occurs naturally in equilibrium, the question becomes what could be done by a social planner to move society towards this social optimum. As is often the case in economics, using tax policy to alter incentives is a viable solution. This paper will look at two alternative tax schemes to achieving a socially optimal outcome.

The first type of tax considered is tolling all roads using marginal cost pricing, consistent with the type of taxes first articulated by Pigou (1920). This type of taxation has received substantial attention in the literature (Smith, 1979; Cole et al., 2006). It has the potential to fully account for all congestion externalities that a driver confers onto other drivers.

Turning back to our basic commuter problem (Figure 1), the marginal external cost conferred onto each other driver is equal to 1/1000 when taking Road A and 1/2000 when taking Road B. The total external cost of an individual's commute is then equal to this marginal cost multiplied by the number of other drivers taking this road. The idea of marginal cost pricing is to charge every driver this rate depending on which road they have taken. This optimal tax scheme is solved in Appendix E and optimal rates are determined to be 14.7 minutes for Road A and 21.7 minutes for Road B. The optimal rates are expressed in terms of minutes and would clearly have to be converted into a monetary value. This could be done by multiplying the optimal toll minutes by a figure equal to the willingness to pay of commuters to avoid an additional minute of commuting time. The actual derivation of this WTP figure may be fairly nuanced in practical applications because the WTP of commuters is likely to vary substantially between individual commuters, meaning a simple average of WTP across the population may not provide an optimal outcome. This is not an insurmountable problem as it mainly a question of data constraints and the tools of cost-benefit analysis may be well suited to solving this problem.

This method of taxation does; however, suffer from a number of other practical problems. First of all, the tax scheme may suffer because, "the principle of marginal cost pricing is single-minded in its pursuit of a minimum-latency flow, and ignores the disutility to network users due to (possibly very large) taxes" (Cole, 2005). The imposition of very large tolls on a large number of roads may significantly alter the incentives for people to drive in general. The model assumes a perfectly inelastic demand for driving and therefore does not take into account that large tolls could lower the total number of commuters and thereby create an end result that is not consistent with the goals of network cost minimization.

Second, the application of this tax system to a complex network may suffer because it would require administering tolls on every road in the network. Utilizing wireless transponders rather than physical toll booths may alleviate this problem somewhat, but the amount of necessary infrastructure is very high, and expecting commuters to rationally cost minimize in a system this complex is probably unrealistic.

This paper will suggest an alternative approach to achieving a socially optimal outcome that overcomes some of these problems. If the network can be modeled quite simply it is possible to have a few targeted tolls which will create a social optimum. Considering once again the example of figure 1, if only Road B is tolled with a cost equivalent to 7 minutes (if WTP to avoid an hour of traffic is found to be $13 then toll would be $1.52) then the Nash Equilibrium will occur when there are 14,667 cars on Road A and 43,333 cars on Road B; which is the social optimum (Proven in Appendix F). This strategy only requires tolling one road rather than both. Furthermore, this toll is significantly smaller than either of the tolls in the marginal cost pricing scheme so it would be less distortionary to the overall incentives.

The approach of targeted tolls is less eloquent than marginal cost pricing because it does not fully internalize the externalities of a drivers actions. When there is perfectly inelastic demand for driving (as has been assumed in the models) then the second method will allow for the same outcome to be achieved with less tolling. Moving to a more complex model with variable traffic volume would impart an added benefit to marginal cost pricing because not only would the distribution of drivers be socially optimal but the total number of commuters who chose to drive would also be socially optimal. This would only be the case if the marginal cost pricing perfectly represented the *true* social cost of driving.

Efficiency Gains Through Road Closures

A method to increase the efficiency of a traffic network, which is unique from a tax scheme, could be to block off some connections in the network. This again seems like an unintuitive solution: shouldn't having fewer roads create more congestion in the system? But as was explored in the case of Braess's Paradox, situations can exist in which adding an additional road can make everyone worse off. Conversely it must be true that removing a road from a network has the potential to make everyone better off. Furthermore, if the question is how to improve overall efficiency of the system we need not make every commuter better off, only make the summation of all travel times lower. This second situation may be possible even if a Braess's Paradox situation does not exist.

Cole et al. points out that "a sufficiently large edge tax effectively removes the edge from the network… Taxes are thus at least as powerful as edge removals" (2006). Although taxes offer greater possibilities for fine tuning the system it may be more practical for a traffic planner to simply block off a certain road. This is not as draconian a policy as it may seem as most cities already institute some form of "local traffic only" policies on some roads. These policies are usually instituted with the goal of calming traffic in a certain neighbourhood but there is no reason why they couldn't be used for general efficiency gains in the network at large.

There have been many papers written on how best to design traffic networks to maximize "robustness" (Stairs, 1968; Jenelius, 2007; Jenelius, 2011). Robustness in this case refers to the networks ability to remain highly functional if any one edge in the network is closed due to unforeseen circumstances (ie. traffic accident). Despite the large amount of literature on this topic there is surprisingly little on possible efficiency gains due to intentional closure of a road. The incorrect intuition that *more roads are better* is once again interfering with analysis. In robustness analysis there is a recognition that how vital a particular road is to a network can take on a range of values (some roads are very important, some have zero contribution); however, this range is incorrectly bounded by zero. It may be that roads exist in a network which have a negative impact on overall efficiency, as has been proven in the extreme case of Braess's Paradox.

Youn et al. successfully identified roads in New York, Boston, and London which could improve network efficiency if they were closed to traffic (2008). Further study in this area could allow traffic planners to better use this potentially powerful tool, which seems to be underutilized in practice, and under-analyzed in the literature.

Conclusion

In Wardrop's first theoretical examination of traffic planning he considered the field to be a "severely practical subject" that had seen little use for theory (1952). Since this time there has been an explosion of research into the game theoretic underpinnings of traffic network planning. It turns out that the topic is extremely nuanced, complex, and often counterintuitive. There is certainly much room for continued research in this field.

Case studies of traffic networks in major cities have shown that there is great potential for efficiency gains by instituting centrally controlled traffic routing (Gonzalez et al., 2007; Youn et al., 2008). These studies demonstrate that tools such as road tolls, and road closures are underused in most metropolitan regions.

The advancement of GPS and wireless data technology coupled with the expanded use of tolling transponders presents opportunities for better network optimization. Ran et al. look at the possibility of continuous, on-the-fly network optimization (1993). This could take the form of a web application which takes into account congestion equations on all routes of the system, as well as current traffic conditions, and network obstructions. The program could then utilize variable tolls on roads which fluctuate to continuously optimize the entire system. Commuters could be made aware of the tolls through wireless access to the application.

Many regions already use variable tolling; for example, having lower tolls on off peak hours. It is not inconceivable that a region could institute tolls which truly capture marginal external costs of each trip, and are able to optimize the system much more efficiently than current practices allow.

Sophisticated economic cost minimization could then be coupled with advances in technology which allow for automated toll collection. Hong Kong pioneered a road tolling system in which cars are equipped with a sensor which emits a personalized signal which is recorded by tolling sensors along the road. Drivers are then sent a bill at the end of the month for their driving tolls. Arnott and Small review the Hong Kong model and claims it to be, “a complete success from an engineering and economic standpoint” (1994). At this time the road tolling in Hong Kong has been discontinued due to public opposition.

Singapore has been able to learn from the successes of the Hong Kong network and implement its own version. The wireless tolling network in Singapore has also been considered successful in reaching its goals, and is looking to update the system by relying on GPS tracking rather than road side tolling sensors.

The field of game theory provides fairly clear solutions regarding how road network efficiency could be improved. A sophisticated understanding of this theory is essential to the design of efficient regional traffic networks. The ability to use new technology to implement network optimization will be the next step in creating road networks which can provide the most efficient transportation possible to expanding urban populations.

Appendix A:

Drive times for Road A and Road B must be equal.

Drive time for A = $44 + (x_A/1000)$ Drive time for B = $30 + (x_B/2000)$

$\therefore 44 + (x_A/1000) = 30 + (x_B/2000)$

$x_A = -14000 + x_B/2$

Total drivers = 58,000 so $58{,}000 = x_A + x_B$ $x_B = 58{,}000 - x_A$

$x_A = -14000 + (58{,}000 - x_A)/2$

$1.5x_A = 15{,}000$ $x_A = 10{,}000$ cars $x_B = 48{,}000$ cars

Drive time for A = 54 Minutes = Drive time for B = 54 Minutes

Appendix B:

Total Drive Time = (Drivers on Road A)•(Drive time on A) + (Drivers on B)•(Drive time on B)

S.T. $X_A + X_B = 58000$

$TDT = X_A(44 + X_A/1000) + X_B(30 + X_B/2000)$

$TDT = 44X_A + X_A^2/1000 + 30(58000 - X_A) + (58000 - X_A)^2/2000)$

$TDT' = 44 + X_A^2/1000 + 30(58000 - X_A) + 1682000 - 58X_A + X_A^2/2000$

$TDT' = 3X_A/1000 - 44 = 0$

$X_A^* = 14{,}667$ cars $X_B^* = 43{,}333$ cars

TC = 3,099,333.3 Minutes Average Drive Time = 53.44 = 53 Minutes 26 Seconds

Appendix C:

Total cost of taking the upper route is: $12 + 4 + x_U/100 = 16 + x_U/100$
Total cost of taking the lower route is: $5 + x_L/100 + 10 = 15 + x_L/100$
We know there are 500 commuters so the restriction of the system is: $x_U + x_L = 500$
It is also known that the travel costs of the two routes must be equal in a Nash Equilibrium.
Solving yields:

$16 + x_U/100 = 15 + x_L/100$
$x_L = x_U + 100$
Restriction: $x_U = 500 - x_L$
$x_L = 500 - x_L + 100$
$x_L = 300$ and $x_U = 200$

Travel time for upper route = 18 minutes Travel time for lower route = 18 minutes

Appendix D:

Suppose a group of commuters must travel from point A to point B each morning. Also suppose there are several possible routes from A to B.
Without central planning the commuters take a combined 500 hours to get from point A to B.
Suppose that a planner can organize the commuters by assigning them specific routes from A to B and that this solution minimizes the combined driving time. Combined commuting time is now reduced to 400 hours.
Price of Anarchy = 500/400 = 1.25

Appendix E:

Marginal Cost Pricing

(Private Cost of Taking Road A) + (Social Cost of Taking Road A) = (Private Cost of Taking Road B) + (Social Cost of Taking Road B)

$44 + (X_A/1000) + X_A(1/1000) = 30 + (X_B/2000) + X_B(1/2000)$
$14 + (X_A/500) = (X_B/1000)$ $58{,}000 = x_A + x_B$
$X_A = 14{,}667$ $X_B = 43{,}333$

Social Cost of A = Toll on A = $X_A(1/1000)$ = 14.7 minutes
Social Cost of B = Toll on B = $X_B(1/2000)$ = 21.7 minutes

Appendix F:

Drive time A = Drive time B + Toll on B
$44 + (X_A/1000) = 30 + (X_B/2000) + T$
At Optimal $X_A = 14{,}667$ $X_B = 43{,}333$
$14 + 14.667 = 21.667 + T$
T = 7 Minutes

Bibliography

Arnott, Richard., Kenneth Small. (1994), The Economics of Traffic Congestion, American Scientist, Vol. 82, 1994.

Awerbuch, Baruch., Azar, Yossi., Epstien, Amir. (2005), The Price of Routing Unsplittable Flow, Symposium On Theory Of Computing Journal, May 2005.

Bergendorff, Pia., Hearn, Donald., Ramana, Motakuri. (1997), Congestion Toll Pricing of Traffic Networks, Lecture Notes in Economics and Mathematical Systems, Springer-Verlag, 1997.

Braess, Dietrich. (1968), On a Paradox of Traffic Planning, *Unternehmensforschung* 12, 1968.

Charnes, A., Cooper, W. (1957), Extremal Principles For Simulating Traffic Flow In A Network., Northwestern Technological Institute And The Transportation Center, And Carnegie Institute of Technology, December 1957.

Cole, Richard., Dodis, Yevgeniy., Roughgarden, Tim. (2005), How Much Can Taxes help Selfish Routing?, Journal of Computer and System Sciences, Vol. 72, 2005.

Gonzalez, Hector., Han, Jiawei., Li, Xiaolei., Myslinska, Margaret., Sondag, John Paul. (2007), Adaptive Fastest Path Computation on a Road Network: A Traffic Mining Approach, VLDB Endowments, 2007.

Jenelius, Erik. (2007), Approaches to Road Network Vulnerability Analysis, Department of Transport and Economics, Royal Institute of Technology, November 2007.

Jenelius, Erik. (2011), Road Network Vulnerability Analysis of Area-Covering Disruptions: A Grid-based Approach With Case Study, Department of Transport and Economics, Royal Institute of Technology, May 2011.

Koutsoupia, Elias., Papadimitriou, Christos. (1999), Worst-case Equilibria, Symposium on Theoretical Aspects of Computer Science, 1999.

Pigou, A.C. (1920), The Economics of Welfare, Macmillan, 1920.

Ran, Bin., Boyce, David., LeBlanc, Larry. (1993), A New Class of Instantaneous Dynamic Optimal Traffic Assignment Models, Operations Research, Vol. 41, No. 1, January 1993.

Rapoport, Amnon., Kugler, Tamar., Dugar, Subhasish., Gisches, Eyran. (2005), Braess Paradox in the Laboratory: An Experimental Study of Route Choice in Traffic Networks with Asymmetric Costs, University of Arizona Press, August 2005.

Smith, Adam. (1776), An Inquiry into the Nature and Causes of the Wealth of Nations, W. Strahan and T. Cadell, London, 1776.

Smith, M.J. (1979), The Marginal Cost Taxation of a Transportation Network, Transportation Res, Part B, 1979.

Stairs, Sonia. (1968), Selecting an Optimal Traffic Network, Journal of Transport Economics and Policy, Vol. 2, No. 2, May 1968.

Wardrop, John Glen. (1952), Some Theoretical Aspects of Road Traffic Research, Road Engineering Division Meeting, January 1952.

Youn, Hyejin., Jeong, Hawoong., Gastner, Michael. (2008), The Price of Anarchy in Transportation Networks: Efficiency and Optimality Control, Korea Advanced Institute of Science and Technology, August 2008.