A Practical Model for Minimizing Waiting Time in a Transit Network

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Introduction

Nowadays urban planners are attempting to improve attraction of public transportation. The most important reason that impedes passengers from choosing public transportation as their trip mode is waiting time in transit's system. According to this fact need of decreasing system's waiting time and transfer coordination is obvious.

Transit waiting time has two major components: boarding waiting time in the first station of the trip and transfer waiting time in the middle stations. Transfer waiting time causes more inconvenience for users as it occurs in the middle of the trip. The first component depends mostly on the headway of the lines and will be reduced by decreasing the headways while the second part varies with several parameters and is more difficult to deal with.

The primary purpose of this research is to create a model to decrease the transfer waiting time of a system with preset headways but to make the model more flexible, we let headways vary in a small range. We put boarding waiting time in the model to minimize both waiting times simultaneously.

In section 2, we reviewed some previous researches on this subject. Section 3 is the representation of the models. In section 4 we described the solution method and section 5 combines unreal and real life examples to show efficiency of the models. Finally conclusions are explained in section 6.

Literature review

Recent studies in transfer optimization have been reviewed and a summary is explained. Ceder et al. (2000) presented a MIP model to maximize the number of simultaneous arrivals of buses from different lines at transfer stations. In this model headways are not preset but they should vary between a maximum and minimum value. The decision variable is the departure time of i-th vehicle in line k (x_{ik}) and the difference between $x_{(i+1)k}$ and x_{ik} should be in the range $[Hmax_k, Hmin_k]$. The travel time of each vehicle is assumed to be deterministic and predictable. Small networks may be solved directly with this model but to solve large scale ones heuristic approaches are used.

Quak (2003) changed the objective function of Ceder's model to minimize passengers' waiting time and solved his model by some changes in Ceder's heuristic model.

Ting (1997) presented two models for transfer coordination. He aimed to minimize cost of system by optimizing headway and dwell time. In his first model travel times are considered to be deterministic and analytical methods are used to solve the model while in the second model travel times are probabilistic and heuristic methods are applied to solve it.

Fleurent et al (2004) introduced three concepts of transfer waiting time: minimum, ideal and maximum transfer waiting time. They made a composite quality index for synchronization and entered it in the cost function as well as other costs. Finally they solved the model using a Lagrangean relaxation and heuristic mechanisms.

Cevallos and Zhao (2006) attempted to shift the existing timetable. Their model used ridership data at all transfer stations and considered randomness of bus arrivals at stations. They solved a network with 80 lines and 255 transfer stations based on genetic algorithm. The results showed 12.1% reduction in the transfer waiting time.

Chung and Shalaby (2008) presented an optimization model to modify the existing timetable of the transit network and exerted extra dwell time in transfer stations. They assumed that buses' arrival time at transfer stations followed a log-normal distribution. This model was solved using genetic algorithm approach.

Shafahi and Khani (2010) proposed two IP models to minimize passengers' transfer waiting time in the network. The variable of the first model is the start time of the first vehicle of each line from the first station. In the second model the stop time of each vehicle at transfer stations is also considered. Shafahi and Khani solved their model by CPLEX package for small networks and Genetic Algorithm for large networks.

Mollanejad (2010) proposed a MIP model to minimize the total waiting time of passengers in transfer nodes of the transit network. The variable of this model is the headway of the lines which is assumed to be uneven. He solved his model using CPLEX package for small networks and simulated annealing algorithm for large networks.

This research attempts to deal with waiting time and presents a mathematical formulation and an efficient solution procedure for transit networks of every size.

Waiting time optimization model

The proposed models for minimizing waiting time in a public transportation system are two mixed integer non-linear programs (MINLP). Objective function of both models is sum of transfer and non-transfer waiting time in the system. Decision variables in the first model are headway and departure time of the first vehicle of each line from the first station. In the second model which is formulated by expanding the first model, an extra stop time is considered for lines in transfer stations, so there would be more successful transfers.

Model assumptions

Some assumptions are made to make the models simpler. The main ones are as below:

- The transit network and fleet size are given.
- Headway of each line is uniform during the planning duration and varies in a small determined range so that fleet size does not change.
- Travel time in each section is given and considered to be constant.
- Travel demand and passengers' chosen routes are given and are independent of systems' characteristics.
- A primary stop time is considered for vehicles in each transfer station, so passengers can get off and on the vehicles. In the second model this value is the output of the model.
- Passengers' transfer time between lines in each transfer station is a given constant value. This time is the minimum possible time that passengers can transfer between vehicles.
- In each transfer station, transferring passengers select the first vehicle of the target line for transferring in order to reduce their transfer waiting time.
- Passengers' average boarding waiting time for each line is assumed to be half of the headway of that line.

Variables and input parameters

The input parameters of the models are:

R: set of all lines in the transit network; i, j and k are line indices, S: set of all transfer stations in the transit network; s is the transfer station index,

S_j: set of all transfer stations in transit network in line j,

$h0_i$: Current headway of line i (in minutes),

t_i^s: Travel time of vehicles from starting point of line i to transfer station s,

dt_i^s: Stopping time of vehicles in line i at transfer station s,

tp_{ij}^s: Number of passengers transferring from line i to line j at transfer station s during the planning duration,

p_i^s: Total number of passengers in line i not transferring at transfer station s during the planning duration (staying aboard),

tt_{ij}^s: required transfer time for passengers transferring from line i to line j at transfer station s. This value is the time needed for a passenger to walk from the vehicle in line i to the vehicle in line j.

p_i: Total passengers of line i including transfer and non-transfer passengers during the planning duration.

Decision variables in the first proposed model are:

$x_k =$ Departure time of the first vehicle of line i from its first station,

h_k = Adjusted headway of line k;

Other variables are:

WT_{ij}^s = minimum waiting time of passengers transferring from line i to line j in transfer station s,

AWt_{ij}^s = average waiting time of passengers transferring from line i to line j in transfer station s,

g_{ij} = greatest common divisor of adjusted headway h_i and h_j,

$y1_{ij}^{s}$, $y2_{ij}^{s}$ = integer variables,
w_{ij}^{s} = binary variable,
Z: objective function; sum of both transfer and non-transfer waiting time in all origins and transfer stations,

Problem formulation

Consider passengers of line i in their origin, we assumed an average boarding waiting time equal to $h_i/2$ for them. So the total boarding waiting time of passengers of line i during planning duration would be equal to $p_i h_i/2$ and the total boarding waiting time in the network would be $\sum_i p_i h_i/2$.

Now let us consider passengers transferring from line i to line j, their transfer waiting time would be equal to (Shafahi and Khani, 2010):

$$WT_{ij}^{s} = \left(x_j + t_j^{s} + dt_j^{s} + y2_{ij}^{s} h_j\right) - \left(x_i + t_i^{s} + tt_{ij}^{s} + y1_{ij}^{s} h_i\right),$$

$$\forall i, j \epsilon R, \forall s \epsilon S \qquad (1)$$

In this equation $(x_i + t_i^{s} + tt_{ij}^{s})$ is the arrival time of the passengers' of the first vehicle of line i and $(x_j + t_j^{s} + dt_j^{s})$ is the departure time of the first vehicle of line j. If the transfer passengers from line i reach the first vehicle of line j the difference between these two sentences would be their transfer waiting time; however, we should consider two cases here: first, the passengers of the first vehicle of line i may miss their transfer to the first vehicle of line j; under this condition, they should wait for the next vehicle of line j. To calculate their waiting time under this circumstance, we added $y2_{ij}^{s} h_j$ to the equation. Second, this transfer waiting time may exceed h_i. As we are calculating minimum transfer waiting time, we should consider passengers of all of the vehicles from line i that have arrived during this period, thus, we added $y1_{ij}^{s} h_i$ to the model.

$y1_{ij}^{s}$ and $y2_{ij}^{s}$ are calculated to gain the minimum waiting time. If headway of line i and j were not equal, the minimum transfer waiting time from line i to line j would not be equal for all the vehicles of line i, so an average waiting time is calculated as below (Shafahi and Khani,2010):

$$AWT_{ij}^{s} = WT_{ij}^{s} + (h_j - g_{ij})/2 , \qquad \forall i, j \epsilon R , \forall s \epsilon S \tag{2}$$

Transfer waiting time for all the passengers changing their line from i to j in the transfer station s would be $AWT_{ij}^{s} tp_{ij}^{s}$ during planning duration. By summing this value for all possible ij-s in transfer station s we can calculate total transfer waiting time in transfer passenger s and by summing this value for all transfer stations in the network we will calculate the total transfer waiting time in the network.

Now we can propose the first model:

$$\text{Minimize } z = \sum_{s}\sum_{i}\sum_{j} tp_{ij}^{s} AWT_{ij}^{s} + \sum_{i} \frac{p_i h_i}{2} \tag{3}$$

$$WT_{ij}^{s} = \left(x_j + t_j^{s} + dt_j^{s} + y2_{ij}^{s} g_{ij}\right) - \left(x_i + t_i^{s} + tt_{ij}^{s} + y1_{ij}^{s} g_{ij}\right) , \qquad \forall i, j \epsilon R , \forall s \epsilon S \tag{4}$$

$$WT_{ij}^{s} \geq 0, \qquad \forall i, j \epsilon R , \forall s \epsilon S \tag{5}$$

$$WT_{ij}^{s} < g_{ij}, \qquad \forall i, j \epsilon R , \forall s \epsilon S \tag{6}$$

$$AWT_{ij}^{s} = WT_{ij}^{s} + (h_j - g_{ij}/2), \qquad \forall i, j \epsilon R , \forall s \epsilon S \tag{7}$$

$$AWT_{ij}^{s} \leq h_j, \qquad \forall i, j \epsilon R , \forall s \epsilon S \tag{8}$$

$$x_k < h_k, \qquad \forall k \epsilon R \tag{9}$$

$$x_k \geq 0, \qquad \forall\ k \epsilon R \tag{10}$$

$$\min h_k \leq h_k \leq \max h_k, \qquad \forall k \epsilon R \tag{11}$$

$$y1_{ij}^{s}: \text{integer}, \qquad \forall i, j \epsilon R , \forall s \epsilon S \tag{12}$$

$$y2_{ij}^{s}: \text{integer}, \qquad \forall i, j \epsilon R , \forall s \epsilon S \tag{13}$$

The objective function is the sum of both transfer and non-transfer waiting time in the network. Constraint 4 calculates the minimum waiting time. Constraint 5 and 6 show the upper and lower bound of the minimum waiting time, respectively. Constraint 7 is the definition of average waiting time. Constraint 8 is the upper bound of average waiting time. Constraint 9 guarantees the practicality of the model. Constraint 11 shows the range in which headway can change to gain the minimum waiting time without causing changes in the frequency. To reduce the solving time an upper bound is considered for $y1_{ij}^{s}$ and $y2_{ij}^{s}$ as below (Shafahi and Khani, 2010):

$$y1_{ij}^{s}.\mathrm{up} = \left[\left|t_j^s - t_i^s\right| + \left(\frac{h_i h_j}{g_{ij}}\right)\right] / g_{ij} \tag{14}$$

$$y2_{ij}^{s}.\mathrm{up} = \left[\left|t_j^s - t_i^s\right| + \left(\frac{h_i h_j}{g_{ij}}\right)\right] / g_{ij} \tag{15}$$

We can also include constraints 16 and 17. As a result, one of the $y1_{ij}^{s}$ or $y2_{ij}^{s}$ would be zero and the other one would have a non-zero value.

$$y1_{ij}^{s} \leq M(W_{ij}^{s}) , \qquad \forall i, j \epsilon R , s \epsilon S \tag{16}$$

$$y2_{ij}^{s} \leq M(1 - W_{ij}^{s}) , \qquad \forall i, j \epsilon R , s \epsilon S \tag{17}$$

As an expansion of the first model we proposed a second model. In the first model we considered a constant stop time for vehicles in the transfer station, in the second model we added the extra stop time (edt_j^s) to the decision variables of the model. Therefore, passengers who miss their transfer with a small gap would have more successful transfers; However, this extra waiting time should not exceed an upper bound, because the aboard passengers travel time would increase and this decreases their tendency toward using public transportation; Moreover, next vehicle would arrive and this extra stop time would be useless.

By entering the new variable to the model some changes are made. First, the definition of minimum transfer waiting time will change as below:

$$\begin{aligned} WT_{ij}^{s} = &\left(x_j + t'^{s}_j + dt_j^s + edt_j^s + y2_{ij}^{s} g_{ij}\right) \\ &- \left(x_i + t'^{s}_i + tt_{ij}^{s} + y1_{ij}^{s} g_{ij}\right) , \end{aligned} \tag{18}$$

$\forall i, j \epsilon R, \forall s \epsilon S$

In which t'^{s}_{j} is the travel time from the first station to the transfer station s and is calculated as below:

$$t'^{s}_{j} = t^{s}_{j} + \sum_{n \epsilon Sb^{s}_{j}} edt^{n}_{j}, \qquad \forall j \epsilon R, s \epsilon S \tag{19}$$

where Sb^{s}_{j} is the set of all transfer stations in the transit network placed in line i before station s.

Upper bounds that are considered for extra waiting time in each transfer station and also total extra waiting time in the network are shown in constraint 20 , 21 respectively.

$$edt^{s}_{j} \leq max\_edt^{s}_{j}, \qquad \forall j \epsilon R, s \epsilon S \tag{20}$$

$$\sum_{s \epsilon S_j} edt^{s}_{j} \leq max\_tedt^{s}_{j}, \qquad \forall j \epsilon R, s \epsilon S \tag{21}$$

Finally we should add the extra travel time of the non-transfer aboard passengers when the vehicle stops for a longer time in transfer stations. This time is equal to $p^{s}_{j} edt^{s}_{j}$, therefore, the total extra travel time in all transfer stations is $\sum_s \sum_j p^{s}_{j} edt^{s}_{j}$.

The proposed model is represented here:

$$\text{Minimize } z = \sum_s \sum_i \sum_j tp^{s}_{ij} AWT^{s}_{ij} + \sum_i \frac{p_i h_i}{2} + \sum_s \sum_j p^{s}_{j} edt^{s}_{j} \tag{22}$$

Constraint 18-19

Constraint 5-13

Constraint 20-21

Genetic Algorithm Approach

Complexity of the model is mostly caused by calculating integer variables $y1^{s}_{ij}$, $y2^{s}_{ij}$ and also g_{ij}. Considering run time and lack of memory, models cannot be solved by common solver packages. On the ground of this fact, we needed a heuristic approach to solve the model. A genetic algorithm is created to solve even very large networks such as urban metropolitans. This approach is briefly described here.

The decision variables of the first model are departure time of the first vehicle of each line from the first station (x_k) and headway of each line (h_k), so chromosomes include these two variables as their genes in the GA. Fitness function is the total waiting time in the network which is the same as the first model's objective function. x_k are defined as double variables which can change in the range $[0, h_k]$ and h_k are defined as integer variables that vary in the range $[hmin_k , hmax_k]$. The first population is created randomly. To create the population of next generations we used linear crossover and mutation operators. Through the linear crossover value of the same genes of two chromosomes are replaced, so the value of genes are still in their range. As crossover operation may result in local optimums, we should exert mutation to search the feasible region completely. Through mutation random values are added to the genes which are chosen randomly. This may cause gene values to exceed their bounds. To prevent this we divided genes' values to their range and replaced them with the remainder of this division to assure that the values are in the feasible region. The other process in the genetic algorithm is the selection of the chromosomes for the next generation. The common selection processes are Roulette wheel and Elitistism. We used both of the procedures in this research.

The final step is to determine termination criteria of the algorithm. The three criteria that we used in this research are:

- The best solution does not change after a given number of iterations.
- The difference between the best and worst solutions in a population is less than a given value, i.e., 1%.
- A maximum number of iterations is reached.

To run the algorithm we need to determine value of algorithm's parameters. One of these parameters is the ratio of crossover-mutation, we gained that by running the algorithm with different ratios for constant number of iterations and investigate the convergence in each of the runs and finally we chose the best rate. The other parameter was number of chromosomes in each generation. We found the best population size following the same methodology as was described for the crossover-mutation ratio. Optimal number of iteration is also determined considering run time and convergence.

To solve the second model we also used the same genetic algorithm with some changes. We should add extra stop time of vehicles in transfer stations to the variables. As all the lines do not pass all the transfer stations, we only considered those i and s for which p_i^s is defined. We replaced this variable in the algorithm with y_i. y_i is defined as integer variables which can change in a specified range. Therefore, the vector formed the genes of the chromosomes includes (x_i, h_i, y_i). The fitness function is the same as objective function of the second model. The other procedures are the same as the first model.

A Real Case Study: City Of Mashhad

Mashhad real life network is used to evaluate the efficiency of model and genetic algorithm approach. Mashhad network is a large network with 139 two ways (278 one way) bus lines and 3618 bus stops of which 841 ones are transfer stations. Figure 1 shows Mashhad network. To estimate the value of the model parameters we performed transit assignment using Optimal Strategy in TransCAD software. Then, we applied genetic algorithm to solve the model of this network. Headways of these lines vary between 2 and 165. We considered a $\pm$10% changes in the headways' value in order not to change the fleet size. Also the upper bound of the edt_i^s was defined as below:

$$edt_i^s.up = \min\left\{4, \frac{h_i}{4}\right\} \tag{23}$$

We estimated GA parameters as described before. We ran the GA for crossover mutation ratios of 0.2, 0.3, 0.4, 0.5, 0.6, 0.7 and 0.8 with 2000 iterations, the optimal crossover mutation ratio was 0.5 considering the GA convergence, then we ran GA with this optimal crossover mutation ratio with population sizes of 10, 20, 30 and the optimal population size was found 20 regarding run time. We also considered 5000 iterations for termination.

Figure 1- Mashhad City bus network

Finally 3 scenarios were solved using GA. The results are shown in table 1.

Table 1
Mashhad transit network scheduling results

scenario	Objective function value	Objective function improvement
No planning	731350	0%
First model	640058	12.5%
Second model	635063	13.2%

By the current condition of the network, waiting time in the whole network during the planning duration is 731350 minutes. By exerting the new condition that is the result of the first model the waiting time would increase to 640058 minutes, this means 12.5% improvement in the system which is equal to 91292 minutes saving time for passengers. Finally by solving the second model the objective

function value would reduce to 635063 minutes. The improvement is 13.16% or 96287 minutes.
Comparing two proposed models, the second model is 0.8% more efficient than the first one; however, we should notice that these results are not the optimal solution.
The trend of the first and second models' objective function improvement versus iterations is shown in figure 2 and 3 respectively.
Finally we can conclude that both models are efficient in reducing waiting time of the passengers in the transit network and GA is an appropriate approach to solve the large scale networks as it does not have limitations for the number of variables and constraints.

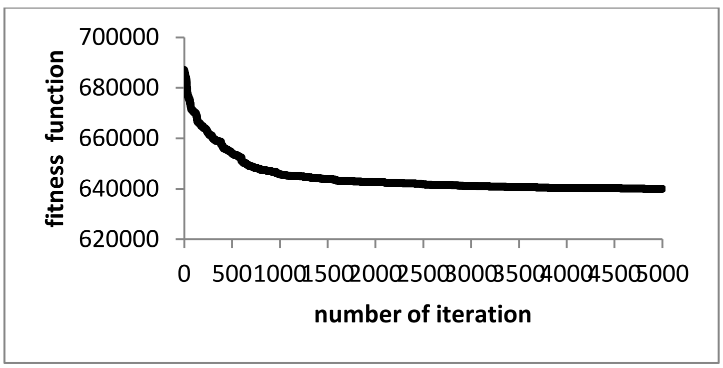

Figure 2- Trend of first model's objective function improvement by number of iterations for Mashhad city transit network using the genetic algorithm

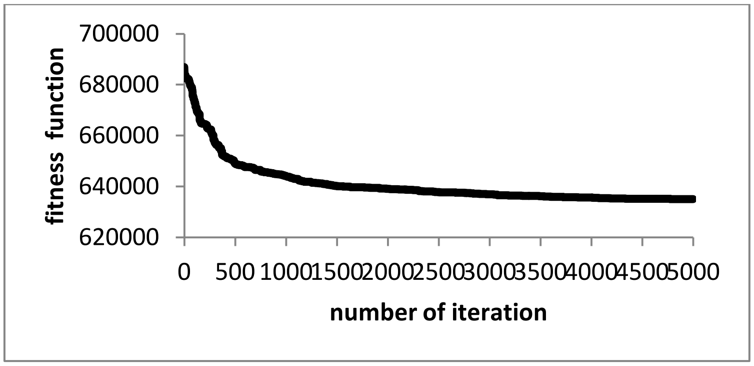

Figure 3- Trend of second model's objective function improvement by number of iterations for Mashhad city transit network using the genetic algorithm

Conclusion

In this paper we represented two models to minimize transfer and non-transfer waiting time in a transit network simultaneously. According to the high number of variables using a heuristic approach to solve the model was necessary and we verified the results with the results of Shafahi and Khani (2010) solution. Finally, we created a genetic algorithm and applied it for a large real life transit network. We made the following conclusions:

- By a small change in headways, there would be a reasonable reduction in the network waiting time; it is noticeable that we assumed uniform headway for each transit line.
- Improvement of the second model in comparing with the first model shows that extra stop time in the transfer station causes more successful transfers and reduces waiting time in the system.
- More significant origins and transfer stations can be weighted in the model based on their numbr of passengers.
- Both of the proposed models reduce the waiting time significantly.

- Finally for ease of the modeling in this research it is assumed that demand of each line is fixed (independent of its characteristics), as an extension of this research, the influence of the changed parameters on demand can be studied. Also stochastic travel time can be considered instead of constant one. Other heuristic approaches or a combination of them can be used to solve the model. Moreover, in this model half of the headway of each line is considered as its average boarding waiting time, it can be replaced with better values.

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