WHAT CONSTITUTES NETWORK ROBUSTNESS? ANALYSIS OF THE CANADIAN BUS TRANSIT SYSTEM

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Introduction and Background

The importance of transit networks in our modern communities is beyond comprehension. In Canada, a total of 3.05 billion passengers have used transit service in 2015 [1]. A significant portion (62.31%) of this population depends on bus transit services as their main mode of mobility, while other transit systems (e.g. ferry, streetcars, rail, and buses) provide service to 37.69% of the population [1]. This significant percentage of bus transit users, relative to other transit systems and not to auto travel, is achieved through 102 bus transit networks operating almost 15,500 buses on 2,300 routes.Therefore, any disruption associated with bus transit network would have severe consequences. Achieving a reliable bus transit operation is a keystone for increasing ridership and reduce car dependency [2,3]

This study adopts Complex Network Theory (CNT) [4]to evaluate the topological characteristics and robustness of bus transit networks in Canada. CNT is a theoretical platform concerned with the study of complex systemsand aims at evaluating the interdependencies between the system's comprising components within different disciplines.Thetheory created remarkable impacts on numerous research fields includingbiology[5], social science [6], psychology [7], power systems [8], and engineering [8].Within the context of transit, CNThas beenutilized in different researchareas such as air traffic [8,9]; rail network [12,13]; and metro (subway) network [14-17]. All these studiesfocused on evaluating the characteristics of the underlying transit networks,which in turn guided the process of improving service performanceby identifying the most influential/critical components within these networks.

In essence, CNT simulates the network through two major components: nodes (vertices) and links (edges). In transit systems, nodes represent transit stations, while the transit corridors between each pair of stations are represented by links. CNT measures that have been recently utilized include Betweenness Centrality [15], Closeness Centrality[18], Clustering Coefficient [12], Average Shortest Path Length [15], Efficiency [17], Robustness [16], and Network Robustness Index [19]. These measures introduced significant contributions to quantifying the characteristics and robustness of diverse transportation networks under different disruption scenarios [20].Closeness Centrality measure [21]quantifies the degree to which a node is adjacent to all other reachable nodes based on the concept of the shortest path. Similarly, Betweenness Centrality [21] evaluates the degree of connection between a node to all other nodes that are not directly connected to it.The Clustering Coefficient[22]determines the degree to which a node in a network tends to cluster with other nodes.The measure of Efficiency[23] provides a holistic indication on the performance level of the network components. It is divided intoLocal Efficiencywhich calculates the efficiency of each node by evaluating the network efficiency with and without that node, andGlobal Efficiency which measures the transition efficiency between all nodes in the network.

Another key network measure is Robustness. Transit network consists of numerous stations and links that transfer millions of passengers. Robustness is generally defined as the ability of a system to fulfill its principle functions when a failure occurs in any component (node or link). The robustness is *"the ability of a system to continue to operate correctly across a wide range of operational conditions and to fail*

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gracefully outside of that range" pp. 17 [24]. While it is definedas *"the degree to which a system is capable of functioning according to its design specifications in the case of serious disruptions"* pp. 3 [25]. Recently,therobust system is defined as the system that *"can maintain its basic functions in the presence of internal and external errors"* pp. 303 [26].

With respect to the applications of CNT in the transit network,theShortest Path Length is used to develop the Robustness Index to analyse the Robustness of the urban transit networks in Beijing, China [27]. Random and targeted attack strategies were applied to evaluate the robustness. In particular, the nodes are removed randomly for the random attack, then the Robustness is measured based on the Average Shortest Path Length in the network. While in the targetted attacks, two different scenarios were applied including the removal of the node with the highest Betweenness Centrality and the removal of the node with the highest Degree.The resilience of transit stations and their impacts on the properties of public transit networks is analysed [28].They implemented different attack scenarios including random failure (random node removal) and targeted attack (removing nodes with the most important operational characteristics).Based on these studies, it could be argued that transit networks donot fit a specific network model (e.g. small world and random), rather the network model emerges from the size of the transit network and the geographical distribution of nodes and links. The literature also highlighted that the dynamic-robustness of transit network under attack are more sensitive to the Betweenness Centrality of the station. While almost all transit networks are not sensitive to random attacks. In this respect, the robustness of transit networks under targeted node removal (Betweenness) varied between smooth and abrupt disruptions, based on network size [27,28].

For the Robustness of bus transit networks, itis expressed, using different metrics, to measure the stress amount (removing or loading a link) that a metro network can sustain before failure (non-operational network) [16]. While, the simulation techniques are used to measure the robustness of metro networks under random failure and targeted attack strategies [17]. In this respect, they have utilized two different measures: disabled route ratio and cost adjustment, to evaluate the effect of a non-operational node on the traffic performance of different metro networks.Similarly, the robustness of 33 metro networks is assessed to study their ability to offer alternative paths under a systemic failure [14].Overall, the developed measures in the literatureare not all consistent in quantifying the robustnessof metro networks.The Clusteringtopological measure represented the robustness of the metro networks, where the higher clustered networks tended to be relatively more robust. Therefore, increasing the robustness depended on increasing the number of transfer stations to offer more alternative paths. It is highlighted that metronetworks exhibit a relatively higher level of robustness under random failures when compared to targeted attacks.While, based on passenger flow, metro networks that accommodate a higher volume of passenger flow are relatively more vulnerable [14,16,17].

The review of CNT applications in transit literature highlights several points. First, there is a clear variation in the topological characteristics and network modelsamongst public transit networks (bus, rail and metro), despite that these could be holistically seen as similar networks. Second, there is a degree of inconsistency with respect to measuring transit network robustness in general. Based on the above, we argue that a more comprehensive approach is required toguide a robustness-based design of bus transit network through assessing transit service at the network-level.As such, we first analyze the topological characteristics of bus transit networks and identify their salient characteristics.Second, the operational characteristics of the bus transit network are subsequently linked, using the degree of association, to the network topological measures. Third, we assess the influence of such characteristics on bus transit network robustness.

Methodology

A dataset of 40 bus transit networks was collected for four groups based on the CUTA classification which classifies bus transit services into groups based on the service area coverage and the corresponding population size [1]. A stratified sampling approach was implemented whereby each group was represented by at least five transit networks as follows: Group 5 (14 networks), Group 4 (13 networks), Group 3 (8 networks), and Group 2 (5 networks). That said, and due to data access restriction, Group 1, which contains 4 transit networks, was not included in the current study.The bus transit networks were modelled as directed graphs, where each bus route represents a link in the network and each station represents a node in the network. It should be noted that links are classified to single and multiple; and nodes are classified into transfer, end, and single nodes. Several CNT measurements (Table 1) are evaluated using Gephi open source software [29] and explained as follows:

The Closeness Centrality (Eq. 1) does not only depend on the number of stations in the bus network, but also takes into account the shortest paths (links) connecting these stations. In this study, the shortest path was represented as the number of steps between bus stations. Since buses operateon fixed routes, there is only one route that represents the shortest path for each bus. The Betweenness Centrality of a station (Eq. 2) is evaluated as the ratio between the number of shortest paths that pass through a specific station to the total number of shortest paths in the entire bus transit network.For Clustering Coefficient (Eq. 3), itis adopted to evaluate thenumber of transfers and alternative routes.The Network Diameter (N_D) is used as an indication ofthe largest number of steps needed to transfer passengers between any pair of stations in the same bus transit network.Hub Index(Eq. 4) is a measure representing nodes in the network with the highest degrees (transfer stations), represented by a larger number of linksthatpass through this station.The Average Path Length (Eq. 5) is the average value of the minimum number of steps between each pair of stations in the same bus transit network.The Graph Density (Eq. 6) is a network property used to determine the degree of dense of the network.It is a relation between the number of single linksand the total number of stations in the bus transit network.The network is considered denser when the number of linksapproaches the maximum possible number of linksin the entire network.

Table 1 CNT measurementsandRobustness metrics used in the current study

Measure	**Equation**	**Eq.**
Closeness	$C_c(i) = \frac{N_{TE} - 1}{\sum_i d(i,j)}$	1
Betweenness	$C_B(i) = \sum_{k \neq i \neq j} \frac{\sigma_{ij}(i)}{\sigma_{kj}}$	2
Clustering	$C(i) = \frac{2L_i}{K_i(K_i - 1)}$	3
Hub Index	$H = \frac{\sum_{N,D>L_{av.}} H_N}{N_{TE}}$	4
Average Path Length	$PL_{av} = \frac{1}{N_{TE}(N_{TE} - 1)} \sum_{i,j=1,N;i \neq j} d_{ij}$	5
Graph Density	$G_D = \frac{L_s}{N_{TE}(N_{TE} - 1)}$	6
Critical Threshold	$f_C = 1 - \frac{1}{\frac{K_{av.}^2}{K_{av.}} - 1}$	7
Robustness Indicator	$R^t = \frac{L_T - N_{TE} - L_m + 1}{N_{Total}}$	8
Robustness Metric	$r^T = \frac{ln(L_T - N_{TE} + 2)}{N_{Total}}$	9
Meshedness Coefficient	$M_g = \frac{L_T - N_{TE} + 1}{2N_{Total} - 5}$	10
Average Degree	$K_{av} = \frac{\sum_{i=1}^{N} K_i}{N_{TE}}$	11
Efficiency	$E_g = \frac{2}{N_{TE}(N_{TE} - 1)} \sum_{i=1}^{N} \sum_{j=1}^{N} \frac{1}{H_{ij}}$	12

This study utilizes the Critical Threshold (f_c),Robustness Indicator (R^t), Robustness Metric (r^T), Meshedness Coefficient (M_g), Network Average Degree (K_{av}), and Global Efficiency (E_g) as robustness measures for the bus transit network.The Critical Threshold (f_c) (Eq. 7) [24] is the threshold of the bus transit network that refers to the fraction of nonoperational stations that yields a completely disconnected network. The mathematical formulation (Eq. 7) is highlighted in Barabási [24].The Robustness Indicator (R^t) (Eq. 8) depends on the number ofalternative routes between stations as an indication of the integrity of the network [14].The Robustness Metric (r^T) (Eq. 9) was advanced by Wang [14] based on the work ofDerrible [14].The Meshedness Coefficient (M_g)(Eq. 10), the Network Average Degree (K_{av}) (Eq. 11), and the Global Efficiency (E_g) (Eq. 12)wereusedas a measure of the network Robustness by Wang [16].

Results

Bus Transit Network Topological Profile

A high value of the Global Closeness Centrality $C_C(G)$ indicates that the bus network has a high adjacency between all stations. Conversely, a low value indicates a weak adjacency for the bus network.As detailed in Table 2, for Group 2, although the number of stations and routes are the highest among all other groups, they are nonetheless distributed overa relatively larger area. Therefore, the probability of the adjacency between these stations is relatively lower, hence the low $C_C(G)$ value (i.e. 0.24). Contrariwise in Group 5, where bus networks are smaller, the number of steps between stations is relatively small, subsequently, the $C_C(G)$ value is higher (i.e. 0.48).

Table 2CNT topological and robustness results for Canadian bus transit networks.

ID	Group ID	Transit Network	N_{TE}	L_T	K_{max}/K_T	$C_C(G)$	$C_B(G)$	$C(G)$	H	PL_{av}	G_D	N_D	f_c	R^t	r^T	M_g	K_{av}	E_g
1	Group 5	Belleville	10	25	0.32	0.51	0.14	0.23	0.24	2.09	0.23	4.00	0.55	0.32	0.077	0.23	5.00	1.18×10^{-4}
2		Cranbrook	9	17	0.41	0.53	0.14	0.13	0.15	2.00	0.22	3.00	0.58	0.32	0.092	0.20	3.78	1.93×10^{-4}
3		Cornwall	6	14	0.50	0.63	0.17	0.00	0.41	1.67	0.33	2.00	0.68	0.17	0.077	0.16	4.67	1.33×10^{-3}
4		Port Alberni	7	21	0.29	0.53	0.20	0.43	0.33	1.98	0.33	4.00	0.21	0.73	0.252	0.88	6.00	5.74×10^{-4}
5		North Bay	4	16	0.50	0.70	0.25	0.00	0.50	1.50	0.50	2.00	0.56	0.13	0.115	0.32	8.00	9.26×10^{-3}
6		Whitehorse	18	51	0.10	0.27	0.18	0.07	0.14	3.79	0.10	8.00	0.02	0.28	0.071	0.36	5.67	5.63×10^{-6}
7		Kitimat	12	29	0.21	0.37	0.18	0.14	0.22	2.83	0.16	6.00	0.08	0.28	0.082	0.27	4.83	4.05×10^{-5}
8		Penticton	30	129	0.11	0.38	0.07	0.24	0.13	3.17	0.10	8.00	0.78	0.75	0.058	0.65	8.60	8.34×10^{-7}
9		Squamish	10	38	0.21	0.39	0.21	0.08	0.26	2.69	0.22	5.00	0.45	0.61	0.189	0.94	7.60	9.18×10^{-5}
10		Fort St. John	10	25	0.20	0.43	0.18	0.16	0.19	2.41	0.21	5.00	0.06	0.59	0.167	0.55	5.00	1.02×10^{-4}
11		Terrace	13	43	0.19	0.36	0.18	0.21	0.23	2.93	0.17	6.00	0.39	0.35	0.081	0.38	6.62	2.81×10^{-5}
12		Leduc	5	13	0.42	0.58	0.27	0.00	0.43	1.80	0.40	3.00	0.01	0.24	0.135	0.31	5.20	2.78×10^{-3}
13		Vernon	9	20	0.40	0.46	0.18	0.00	0.29	2.28	0.19	4.00	0.76	0.21	0.088	0.23	4.44	1.69×10^{-4}
14		Welland	8	23	0.44	0.59	0.13	0.28	0.24	1.79	0.29	3.00	0.81	0.31	0.098	0.30	5.75	3.57×10^{-4}
Mean			10.79	33.14	0.31	0.48	0.18	0.14	0.27	2.35	0.25	4.50	0.42	0.38	0.113	0.41	5.80	1.08×10^{-3}
15	Group 4	Chilliwack	17	50	0.16	0.37	0.12	0.12	0.19	2.84	0.15	6.00	0.53	0.51	0.073	0.37	5.88	9.53×10^{-6}
16		Prince George	21	85	0.15	0.36	0.10	0.29	0.18	2.83	0.15	7.00	0.70	0.98	0.097	0.80	8.10	4.00×10^{-6}
17		Brantford	12	31	0.36	0.55	0.09	0.29	0.25	1.89	0.21	3.00	0.79	0.35	0.063	0.22	5.18	6.09×10^{-5}
18		Saint John	32	99	0.12	0.26	0.10	0.08	0.12	3.97	0.07	9.00	0.63	0.41	0.042	0.34	6.19	5.11×10^{-7}
19		Kingston	23	82	0.17	0.49	0.08	0.21	0.17	2.34	0.13	5.00	0.81	0.77	0.079	0.61	7.65	3.34×10^{-6}
20		Brandon	12	34	0.27	0.47	0.12	0.24	0.25	2.18	0.21	4.00	0.68	0.43	0.079	0.31	5.68	5.26×10^{-5}
21		Barrie	43	96	0.16	0.27	0.04	0.14	0.08	3.27	0.05	8.00	0.86	0.79	0.070	0.50	4.47	1.88×10^{-7}
22		Lethbridge	13	34	0.27	0.43	0.13	0.10	0.24	2.42	0.17	4.00	0.58	0.31	0.070	0.26	5.23	3.39×10^{-5}
23		Milton	10	26	0.39	0.52	0.13	0.07	0.28	2.02	0.22	3.00	0.74	0.23	0.061	0.19	5.20	1.22×10^{-4}
24		Nanaimo	14	70	0.17	0.46	0.11	0.32	0.22	2.26	0.23	5.00	0.71	0.40	0.049	0.35	10.00	2.67×10^{-5}
25		Kamloops	19	57	0.16	0.35	0.12	0.23	0.18	3.01	0.13	6.00	0.66	0.42	0.059	0.33	6.00	5.68×10^{-6}
26		Thunder Bay	20	73	0.14	0.32	0.13	0.23	0.17	3.27	0.14	7.00	0.62	0.47	0.056	0.39	7.30	4.23×10^{-6}
27		Whistler	8	20	0.40	0.45	0.15	0.00	0.28	2.02	0.23	4.00	0.68	0.26	0.098	0.27	5.00	3.16×10^{-4}
Mean			18.77	58.23	0.22	0.41	0.11	0.18	0.20	2.64	0.16	5.46	0.69	0.49	0.069	0.38	6.30	4.92×10^{-5}
28	Group 3	Burlington	28	111	0.14	0.35	0.08	0.24	0.14	3.01	0.10	6.00	0.80	0.36	0.031	0.30	7.93	1.16×10^{-6}
29		Windsor	22	76	0.13	0.38	0.09	0.22	0.16	2.76	0.14	5.00	0.50	0.44	0.042	0.29	6.91	3.39×10^{-6}
30		Regina	38	147	0.16	0.36	0.05	0.18	0.12	2.82	0.08	6.00	0.88	0.76	0.050	0.60	7.74	3.58×10^{-7}
31		Niagara Region	10	31	0.26	0.49	0.14	0.13	0.25	2.12	0.24	4.00	0.56	0.32	0.077	0.29	6.20	1.16×10^{-4}
32		Oakville	30	132	0.17	0.36	0.07	0.19	0.14	2.89	0.10	6.00	0.90	0.31	0.026	0.29	8.80	9.16×10^{-7}
33		London	65	217	0.07	0.27	0.04	0.10	0.09	3.73	0.04	9.00	0.76	0.48	0.021	0.32	6.68	3.10×10^{-8}
34		Victoria	103	452	0.05	0.22	0.04	0.12	0.07	4.66	0.03	10.00	0.84	0.80	0.026	0.79	8.78	3.89×10^{-9}
35		Saskatoon	52	160	0.16	0.36	0.03	0.18	0.09	3.12	0.05	6.00	0.92	0.53	0.036	0.42	6.15	9.10×10^{-8}
Mean			43.50	165.75	0.14	0.35	0.07	0.17	0.13	3.14	0.10	6.50	0.77	0.50	0.039	0.41	7.40	1.53×10^{-5}
36	Group 2	Brampton	115	407	0.05	0.19	0.04	0.11	0.06	5.37	0.02	13.00	0.87	0.47	0.014	0.36	7.08	2.17×10^{-9}
37		Durham Region	85	273	0.05	0.26	0.04	0.15	0.08	4.49	0.03	10.00	0.75	0.46	0.016	0.28	6.42	8.75×10^{-9}
38		Ottawa	252	1338	0.04	0.20	0.01	0.17	0.03	4.78	0.01	11.00	0.96	0.83	0.012	0.96	10.62	1.05×10^{-10}
39		Hamilton	55	148	0.08	0.27	0.07	0.13	0.07	5.95	0.04	15.00	0.69	0.19	0.014	0.15	5.46	3.81×10^{-8}
40		Waterloo	65	206	0.06	0.27	0.05	0.15	0.10	4.19	0.05	10.00	0.63	0.42	0.020	0.29	6.34	2.76×10^{-8}
Mean			114.40	474.40	0.06	0.24	0.04	0.14	0.07	4.96	0.03	11.80	0.78	0.47	0.015	0.41	7.18	1.53×10^{-8}

Results of the Global Betweenness Centrality $C_B(G)$ show that Group 5 has the highest $C_B(G)$ mean value of 0.18, and this value decreases to 0.11, 0.07, and 0.04 for Groups 4, 3, and 2, respectively. This indicates that few stations in Group 5 act as the main nodes (terminals) in the network and the disruption of any of those station imposes a significant impact on the network. While the opposite is true for Group 2, which indicates that there are several shortest paths alternatives between stations.A high Global Clustering Coefficient *C(G)* value indicates that the neighbours of each station in the network are relatively more interconnected, which provide a larger number of alternative routes for bus users to travel between stations. In contrast, a lower value indicates weak interconnectivity of the bus transit network, and subsequently difficulties in the movements between stations. The mean Clustering values range between 0.14, 0.18, 0.17, and 0.14 for Groups 2, 3, 4, and 5, respectively.Considering the entire dataset, the highest Hub Index (*H*) value is 0.50, while the lowest is 0.03. In general, Group 5 has the highest mean value for the Hub Index of 0.27 and this value decreases for other groups reaching 0.20 for Group 4, 0.13 for Group 3, and 0.07 for Group 2.The Average Path Length (PL_{av}) is affected directly by the size of the bus network, where larger networks, such as Groups 2 and 3, have higher PL_{av}, while smaller networks, such as Groups 4 and 5 possess smaller PL_{av}values, with an overall range between 1.50 and 5.95 steps. A Graph Density (G_D) highest valueis 0.50 in Group 5 and the lowest value is 0.01 in Group 2.This conforms to practice where smaller networks have a small number of stations and require a few routes to connect them. Therefore, the available number of routes is probably close to the maximum possible number of routes, whilethe opposite is true for large networks.The values of the Network Diameter (N_D) vary between 15 and 2 steps, respectively. Where, Group 2 has the highest mean value for N_D(N_D=11.80), this is followed by Group 3 (N_D=6.50), Group 4 (N_D=5.46), and Group 5 (N_D=4.50).

Degree of Association Between the Bus Transit Network Parameters and Network Topology

All previous measures introduced a general description of the topological profile associated with different bus transit networks. This section further discusses the relationships between six transit network topology measures and transit network operational characteristics with emphasis on the number of transfer/end stations (N_{TE}). Figure 1shows the relationships between the number of transfer/end stations (N_{TE}) and six topological measures. The figure clearly indicates the existence of a higher degree of association between N_{TE} and their corresponding network topology measures. It should be noted that there is no direct relationship observed from the data between the Clustering Coefficient and N_{TE}. This is due to the fact that the Clustering Coefficient depends directly on the degree of connection between the neighbours of each station regardless of the number of transfer/end stations within the bus network.

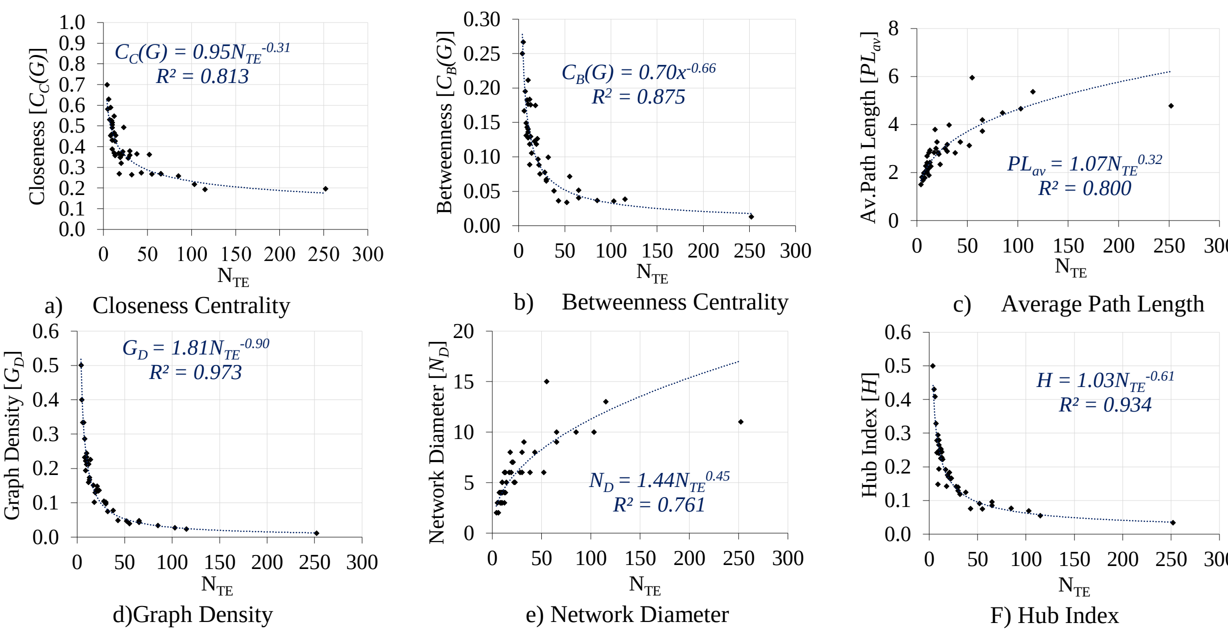

Figure 1 The association between bus network parameters and complex network measures

Robustness of Bus Transit Networks

The robustness of bus transit network in this study is utilized as an indication of the extent to which a bus transit network is capable of maintaining its function in transporting passengerswhen a station and/or a route becomes non-operational. A high robustness value indicates a higher ability of the bus transit network to remain operational underdisruption.With respect to the robustness of transit networks based on CNT models, previous studies reported the dynamic-robustness and the resilience of the network [27,28,30]. This approach is mainly based on node removal strategies. In our results, we depend on the network-level static robustness measures that are based on the topological profile of the entire network.

The results of robustness measures are listed in Table 2. Results of The Critical Threshold (f_c) range between 0.960.01.The results indicate that Group 2 has the highest mean f_c value of 0.78, and this value decreases to 0.77, 0.69, and 0.42 for Groups 3, 4, and 5 respectively.Results of Robustness Indicator (R^t) range from0.98to 0.13. The mean R^t values are 0.47, 0.50, 0.49, and 0.38 for Groups 2, 3, 4, and 5, respectively. Where Group 5 has the lowest value and Group 4 has the highest value for R^t in the dataset. For the Robustness Metric (r^T), the highest value is 0.252 while the lowest valueis 0.012. Generally, bus networks in Group 5 have the highest r^Tmean value of 0.113 and this value decreases gradually to 0.069, 0.039, and 0.015 for Group 4, Group 3, and Group 2, respectively.In terms of the Meshedness Coefficient (M_g), the highest M_g is 0.96 in Group 5 and the lowest value is 0.15 in Group 2. The mean M_g values are 0.41, 0.38, 0.41, and 0.41 for Groups 5, 4, 3, and 2, respectively. The lowest and highest values of Network Average Degree vary between 3.78 and 10.62.For the Network Average Degree (K_{av}), Group 5 has the lowest mean value of 5.797, this is followed by Group 4 (6.30), Group 3 (7.40), and Group 2 (7.18). Finally, the Global Efficiency (E_g) values range between$1.05\times10^{-10}$and 9.26×10^{-3}. In general, Group 5 has the highest mean value of 1.08×10^{-3}, which decreases to $4.92\times10^{-5}$for Group 4, $1.53\times10^{-5}$for Group 3, and$1.53\times10^{-8}$for Group 2. The E_g values could be seen as negatively associated with the size of the bus network.

The Robustness of Bus Transit Network: A Discussion

In this study, we argue that the robustness of bus transit networks should be evaluated based on network topology measures, which provide details on the relationships and arrangements of bus stationsand routes within the transit network.In fact, from the transit planning perspective, both the topology and robustness measures could provide informative indicationsofthe performance of the bus transit network. That said, the results have highlighted that robustness and topology measures vary significantly in their applications and outcomes. This is mainly attributed to the fact that each measure includes a unique set of parameters, which impact the resulted values.Accordingly, we argue that informative robustness and topology measures of bus transit networks should reflect the practice of bus transit networks design and operation.However, such a reflection is not currently well established in the literature. For example, in existing numerical models, any station featuring more than one transit route ($L_i>2$) is considered as a transfer station (N_{TE}).Therefore, models that are based only on N_{TE} cannot account for the variation between transfer stations with a relatively higher number of connected bus routes and those witha lower number.Both types have the same impact on estimating several measures, which indeed does not reflect real-world practice. Therefore, in our discussion of the practical implications of bus transit topology and robustness measures, we focus on measures that provide comprehensive indications to transit planners and policymakers, while highlighting the limitation of some models.

Similar to topology measures, results of the robustness measures show a noticeable variation. Furthermore, the values of robustness for the same bus network differs significantly between measures. For example, some robustness measures including M_G, R^t, r^T, and E_g are mainly based on bus network parameters such as the number of transfer stations and links, while the K_{av} is based on the degree centrality of the bus network and the f_c depends on the degree distribution and its second moment.As a result, we argue that some of these measures cannot be jointly used as indications of the robustness of bus transit networks. We further argue that robustness measures (R^t, r^T, and M_g),which are based solely on the bus network parameters (N_{TE} *and* L_T) are limited due to their static nature as they fall to account for different topologies associated with bus transit networks. Moreover, robustness measures (K_{av} *and* E_g) that are partially based on bus network parameters (N_{TE} *and* L_T) are more comprehensive as they account for the variation in the network topology.

In this respect, two measures should be inspected jointly as they provide indications on both the total number of N_{TE} and the number of links associated with each station. These are the Global Betweenness Centrality ($C_B(G)$) and the Hub Index (H). First, the value of the $C_B(G)$ providesa direct indication to assess bus transit network. An increase in the $C_B(G)$value refers to the existence of stations that lie on the almost all shortest paths in the network, and hence these stations have a direct impact on the operation of the bus transit network. Furthermore, this provides indications on the efficiency of the bus transit network, as it highlights stations with higher impacts, if disrupted, on the network. Second, the value of the Hub Index measure (H) provides a clear indication on the centralization of the bus transit network, where high value is associated with networks that include onlya few hubs. Adecrease in the H could be seen as the lack of dominant stations in the entire network, and therefore, the bus transit networkis more robust against failure. Therefore, it can be argued that transit networks which have relatively lower$C_B(G)$ *a*nd lower *H*values been relatively more efficient. These two measures could be seen from transit planners' perspective as indications of the connectivity of a node (high $C_B(i)$) and the lack of central dominating stations in the network (low $C_B(G)$) and H).These results confirm previous literature which concluded that targeting stations with higher Betweenness Centrality and/or Hub Indexvalues pose significant impacts on transit network robustness [27,30].Furthermore, we argue that the Critical Threshold (f_c)is relatively the most comprehensive robustnessmeasurefor bus transit network as it is based on the degree distribution of the links in the bus network. This providesquantified evidence on the importance of transfer stations and the percentage of stations that could be removed until the network is unconnected. Although these results are based on static network-level measures, they confirm the results of dynamic-robustness models developed in previous studies[9,27,30].

Conclusion

This study utilizes Complex Network Theory (CNT) to quantify the topological profile and the robustness of bus transit networks as well as their linkage. This is mainly carried out to better inform transit planners and policy-makers on avenues to enhance the robustness of bus transit networks.Towards that end, different network-level topological measuresare quantified for 40 Canadian bus transit networksthat represent five groups, each with a distinct operational profile[1]. As it relates to topological measures, bus transit networks in Group 5 (the lowest population size, fleet size, and coverage area) show the highest value of Closeness, Clustering, Hub Index, Graph Density and Global Betweenness. In addition, these networks have the lowest Average Path Length, Network Diameter, and Clustering. In contrast, bus transit networks in Group 2 (the highest population size, fleet size, and coverage area) have the highest Betweenness, Average Path Length, and Network Diameter, while having the lowest Closeness, Hub Index, and Graph Density.The results indicate that there is a noticeable association between$N_{TE}$and the different topologicalmeasures. Where, $N_{TE}$is directly proportional to Average Path Length, and Network Diameter, while it is inversely proportional to the Closeness, Hub Index, Betweenness, and Graph Density. At the network-levelassessment, it is argued that networks featuring low Global Betweenness Centrality and low Hub Index are well distributed and offer numerous alternatives of movement with the network. In addition, and at the node-level assessment, it is argued that nodes with higher Local Betweenness Centrality are well connected and these repents key building blocks of the network.

The robustness of bus transit networks is assessed using six measures. The results show a significant variance between robustness measures, highlighting that the utilization of these measures should be carefully investigated. The study concludes that robustness measures that are solely dependent on the number of transfer/end stations and the number of links inherit some limitations. These are mainly attributed to its inability to capture the varied characteristics of transfer stations. Overall, networks in

Group 2 show higher robustness values, realized through the availability of multiple alternative routes between station. While networks in Group 5 (which mainly feature a hub & spoke network shape) are less robust.

It is important to note that the current study has some limitations. Aside from the fact that the current study did not represent transit networks in Group 1 due to data limitation, the study did not account for some transit operation data such as the variation in timetables and passenger capacity. Such transit operation data would require a more dynamic modelling approach to consider the temporal variation in link weight and station degree. Nonetheless, several recommendations could be inferred from the present study.

For transit planners, the study argues that Global Betweenness Centrality and Hub Index measures are effective indicationsto the distribution of bus transit network components (nodes and links). Where lowerGlobal Betweenness Centrality value associated with lower Hub Index valueis anindication of a well-connected and decentralized network. The study also argues that the Critical Threshold (f_c) is the most comprehensive topology-based measure to evaluate bus transit network robustness, as other measures are mainly based on the static values of network parameters. This measure provides indications on the level of robustness associated with specific bus transit networktopologies, which could be used to guide future network reconfiguration and expansion. While for policy-makers, we recommend the inclusion of network-level measures in the design process of the bus transit network, which proved to be effective in capturing the robustness of transit services.Finally, it is important to note that the developed models could be readily implemented across different context for a holistic evaluation of other transit system network topology and robustness assessment.

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