COMPARING TWO COMPETING MODELS TO MEASURE ANNUAL PRODUCTIVITY GROWTH RATES IN THE CANADIAN RAILWAY INDUSTRY[1]

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Introduction

There are many ways to measure productivity, though in recent years, two approaches have dominated the calculation of productivity changes in various situations: the Malmquist and the Luenberger approaches. In this paper we compared those two approaches from both a theoretical and an empirical perspective. Because of data limitations, in our study the Canadian Railway Industry is limited to Canadian Pacific (CP) and Canadian National (CN). Although there are other smaller railways in Canada, these two big companies constitute the major part of the Canadian Railway Industry.

Two main articles in the literature compare the Luenberger and the Malmquist productivity models.
Boussemart et al (2003) prove that the logarithm of the Malmquist input productivity index is twice a linear approximation of (minus) the Luenberger productivity indicator. Their empirical results studying the productivity index for 20 OECD countries over the 1974 to 1997 period using Malmquist and Luenberger approaches show that the annual productivity growth rates using Malmquist approach were twice the ones when using a Luenberger model. However, their calculations were based on a Data Envelopment Analysis that uses linear approximation rather than direct calculations of the theoretical relationships and required data from a large number of production units over several years, to permit estimation of the underlying production frontier for the group. Thus, their methodology is applicable only to the study of productivity growth at the national level, and cannot be used for individual firms. Moreover, their theoretical formulation assumes constant returns to scale, which is not realistic for the railway industry.

In an exposita note, Balk et al (2008) attempt to show the exact mathematical relationship between Luenberger and Malmquist productivity indexes, rather than their linear approximations. However, their comparison was based on a specific scenario where the Luenberger productivity index is calculated in a case where there is only an output expansion. In this particular configuration, they showed that there must be an exact match between both indexes if some technical assumptions are made on the technology. The comparison was therefore limited to a particular case and no empirical study using the exact theoretical relationships was provided. The two studies suffer from a conceptual weakness. We present a study of the exact relationship between the two methods from an empirical perspective.

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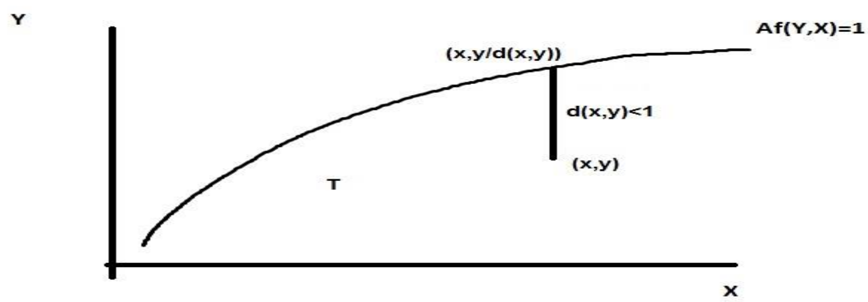
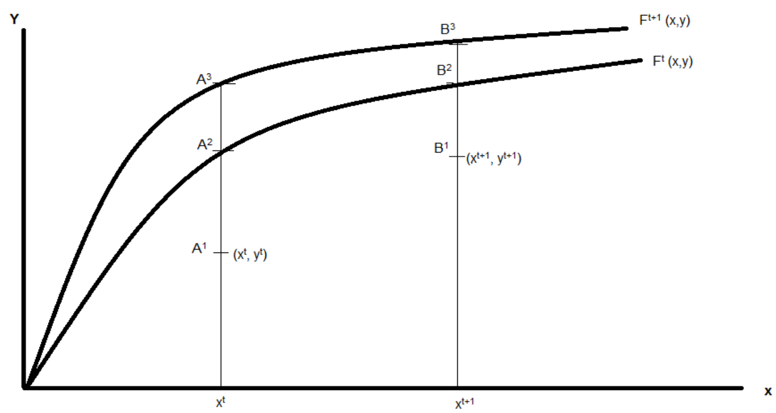
Malmquist Productivity Index:

The concept of distance is central to all productivity models. In this approach, distance are measured *vertically* as the result of increasing output (y) while inputs (x) are constant:



In the above figure, *Y* represents the output vector and *X* the input vector. The technology curve (*Af(Y,X)=1)* represents the technical constraint, and *T* the domain of feasible output and input combinations.
Productivity is measured as the *vertical* distance between two technology curves. In the figure below, we assume that the technology has improved so that the technology curve shifted from F^t to F^{t+1}. In the meantime the combination of outputs and inputs shifted from $A^1=(x^t, y^t)$ to $B^1=(x^{t+1}, y^{t+1})$.



By using this *vertical* distance framework, in 1982,C.C.D made the micro-economic connection between a firm's profit maximization program and a relationship that measures its productivity growth in a period of time (between t and t+1), so that productivity is a direct consequence of profit maximization conditions.
In 2010, Diewert and Fox proposed an extension of the standard C.C.D approach by introducing imperfect competition.

Using the analytical approach proposed by Chambers (2002) and Hudgins and Primont (2004), we have re-formulated the Diewert & Fox extension of the traditional CCD model, to derive a fully deterministic productivity equation that allows for imperfect competition and non-constant returns to scale.

Technically speaking, at any given time*s*, the railway will solve the following constrained optimization problem to maximize profits:

$$\begin{cases} \max_{y^s,\ x^s} \sum_i P_i^s(y_i^s) y_i^s - \sum_n w_n^s x_n^s \\ constraint: \quad (x^s, y^s) \in T \end{cases}$$

At each time*s*, the railway will choose optimal levels of outputs and inputs under the technology constraint which can also be expressed in terms of distance to the technical frontier. This distance depends on the mathematical specification given to the distance function which is directly connected to the mathematical specification that is given to the technology curve. Therefore, the measurement of productivity ,which is the distance between to two technology curves, depends on the optimal levels of output and input, and also on the way distance is measured.

Under the assumption that the technology curve is a trans-log function, solving the above optimization problem at time*t* and *t+1* and measuring the *vertical* distance between the two technology curves at the optimal combinations of outputs and inputs leads to the following Malmquist Productivity Index (MPI):

$$\begin{aligned} TFP \quad = \quad & \frac{1}{2}\sum_{i=1}^{I}\left[\frac{P_i^{t-1}\left(1-m_i^{t-1}\right)y_i^{t-1}}{\sum_{i=1}^{I} P_i^{t-1}\left(I-m_i^{t-1}\right)y_i^{t-1}} + \frac{P_i^{t}\left(1-m_i^{t}\right)y_i^{t}}{\sum_{i=1}^{I} P_i^{t}\left(I-m_i^{t}\right)y_i^{t}}\right]\left(lny_i^t - lny_i^{t-1}\right) \\ & - \frac{1}{2}\sum_{n=1}^{N}\left[\frac{w_n^{t-1}x_n^{t-1}}{\sum_{i=1}^{I} P_i^{t-1}\left(I-m_i^{t-1}\right)y_i^{t-1}} + \frac{w_n^{t}x_n^{t}}{\sum_{i=1}^{I} P_i^{t}\left(I-m_i^{t}\right)y_i^{t}}\right]\left(lnx_n^t - lnx_n^{t-1}\right) \end{aligned}$$

where:

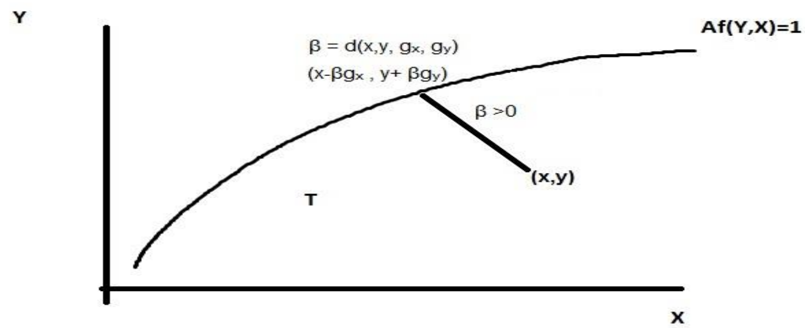
TFPis the ratio of Total Factor Productivity between time periods t and t-1;
ln y_i^t and ln y_i^{t-1} are the logarithms of the i^{th} output in periods t and t-1;
ln x_n^t and ln x_n^{t-1} are the logarithms of n^{th} input in periods t and t-1;
w_n^t and w_n^{t-1} are the prices of the n^{th} input in periods t and t-1;
P_i^t and P_i^{t-1} are the prices of the i^{th} output in periods t and t-1; and
m_i^t and $m_i^{t-1}$are the markup factors by which marginal product exceeds factor cost for the i^{th} output in periods t and t-1.

The above formula tells us that productivity is the difference between the weighted average of output changes and the weighted average of input changes. Imperfect competition is captured by markup factors *m* and returns to scale are not set to be constant since it was not assumed that the trans-log technology exhibits constant returns to scale. Although it looks no more than a classical volume index, it is important to note that mathematical assumptions are behind weighting schemes of output and input changes. In the next section we will see that different mathematical assumptions lead to different weighting schemes. The question is how much the measurement of productivity is sensitive to these different technical assumptions.

Luenberger Productivity Index:

As opposed to the previous method, the Luenberger approach is additive and assumes that distance is the result of not only increasing output level, but also decreasing input level.

Graphically it can be represented as follows:

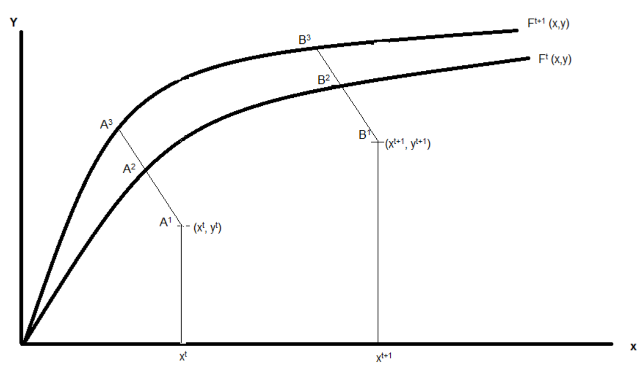


Very often in the literature, fixed vectors g_{y^t} and g_{x^t} are set the following way:

$$\forall t, \quad g_{y^t} = y^t$$

$$\forall t, \quad g_{x^t} = x^t$$

Productivity gains are in this case measured as the *oblique* distance between two technology curves:



After solving the same optimization problem as we did in the Malmquist approach along with the same assumptions – except the mathematical assumption on the distance metric – we find the following Luenberger Productivity Index:

$$TFP = \ ln\left[\frac{\tilde{P}_t^{t+1}.\, y^{t+1}}{\tilde{P}_t^{t+1}.\, y^t}\right] - ln\left[\frac{\widetilde{w}_t^{t+1}.\, x^{t+1}}{\widetilde{w}_t^{t+1}.\, x^t}\right]$$

$\tilde{P}_t^{t+1} = \frac{1}{2}[\tilde{P}_t^{t+1} + \tilde{P}_t^t]$; $\tilde{P}_t^t = \frac{P^t.(1-m^t)}{P^t.(1-m^t)+w^t.x^t}$; $\widetilde{w}_t^{t+1} = \frac{1}{2}[\widetilde{w}_t^{t+1} + \widetilde{w}_t^t]$; $\widetilde{w}_t^t = \frac{w^t}{P^t.(1-m^t)+w^t.x^t}$

where all symbols retain the same meanings as for the Malmquist approach.

DATA USED

Output Quantities

For estimation of the Malmquist and Luenberger productivity indices this study uses a disaggregated measure of outputs, comprising the revenue tonne-miles performed by CN and CP of 14 commodity groups based on CN and CP's lines of business.

Quantity indices are derived for each of the 14 commodity groups from 1992 to 2014, with the value of the 1992 RTM of each group set to 1.

Output Prices

The output price for each commodity group is derived as the average revenue obtained by CN or CP for that commodity group divided by the RTM performed of that group. Revenue shares for each commodity group are obtained directly from the same database.

Input Quantities

The inputs used in this study follow the classical KLEMS (capital, labour, energy, materials, services) used in most productivity studies.

Labour

Annual Labour quantities are derived from the annual reports submitted to the Agency by CN and CP on compensation and actual hours worked for each of 79 categories of labour.For each railway, a labour quantity index is constructed of the annual total labour hours worked, with the 1992 value set to 1. Before constructing the quantity index, the total hours worked are reduced by the labour hours spent on capital projects, as those hours are included in the capital quantities.

Fuel

Annual fuel quantities are derived from the annual reports submitted to the Agency by CN and CP on the total litres of fuel consumed in rail operations in each province. For each railway, a fuel quantity index is constructed of the annual total fuel consumed, with the 1992 value set to 1

Capital

This study constructed quantity indices for four categories of capital assets: land, infrastructure, owned equipment, and rented and leased equipment. Both railways use the perpetual inventory method to keep track of the value of reasonably homogeneous groups (for example, locomotives, freight cars, rail, ballast, etc.) of their owned capital assets, in each year. For each class of assets, the railways record the gross value of the capital stock at the beginning of the year, the additions to capital assets (new investments) during the year, the reductions to capital assets (retirements and depreciation) during the year, and the gross value at the end of the year.

Because land is not a depreciable asset, the gross amounts for land (account 101 in the UCA) are deflated to constant 1992 dollars using the GDP deflator, and a quantity index of land is constructed from the resulting constant dollar series by setting the 1992 value to 1.

Materials

Materials comprise thousands of parts and supplies which, as with capital, makes it impractical to develop an aggregate quantity index based on the quantities of the material items. Instead, a quantity index for materials is developed by dividing annual expenditures on materials by a materials price index approved by the Agency for use for regulatory purposes, then converting the resulting quantity series into an index with 1992 value set to 1. Material expenses are developed from Expense Reports, which excludes the materials used in capital projects.

Services
Services include all other expenses incurred by the railway excluding labour, fuel, capital and materials. It includes engineering, legal, accounting, consulting, contracting, and other services purchased by the railway. A quantity index for services is developed by dividing annual expenditures on services by the GDP deflator, then converting the resulting quantity series into an index with 1992 value set to 1. Services expenses are developed from Expense Reports and include no capital-related expenses.

Input Prices

Labour
The annual price of labour is calculated as the average dollar of employee compensation per hour worked. Total compensation for employees includes salaries and wages, wage-related benefits (paid vacations, bonuses, share-purchase plans, etc.), employment benefits (CPP, QPP, EI, health & welfare payments, etc.), stock-based compensation, and pension benefits. A labour price index is constructed from the annual price series by setting the price for 1992 to 1.

Fuel
The annual price of fuel is calculated as the fuel price per litre, which details the litres of fuel consumed and corresponding fuel expense incurred by Province, submitted annually to the Agency by each railway. A fuel price index is constructed from the annual price series by setting the price for 1992 to 1.

Capital
Unlike, say fuel, a capital item is not fully consumed in the provision of a transportation service. A capital item is long-lived, and is best thought of as providing a flow of services over its useful life. The price of the service flow provided by a capital item in any given year is therefore very different from the purchase price of the capital item, and is related to many factors including the opportunity cost of capital, the rate of depreciation of the asset, property taxes, investment tax credits, and other factors.

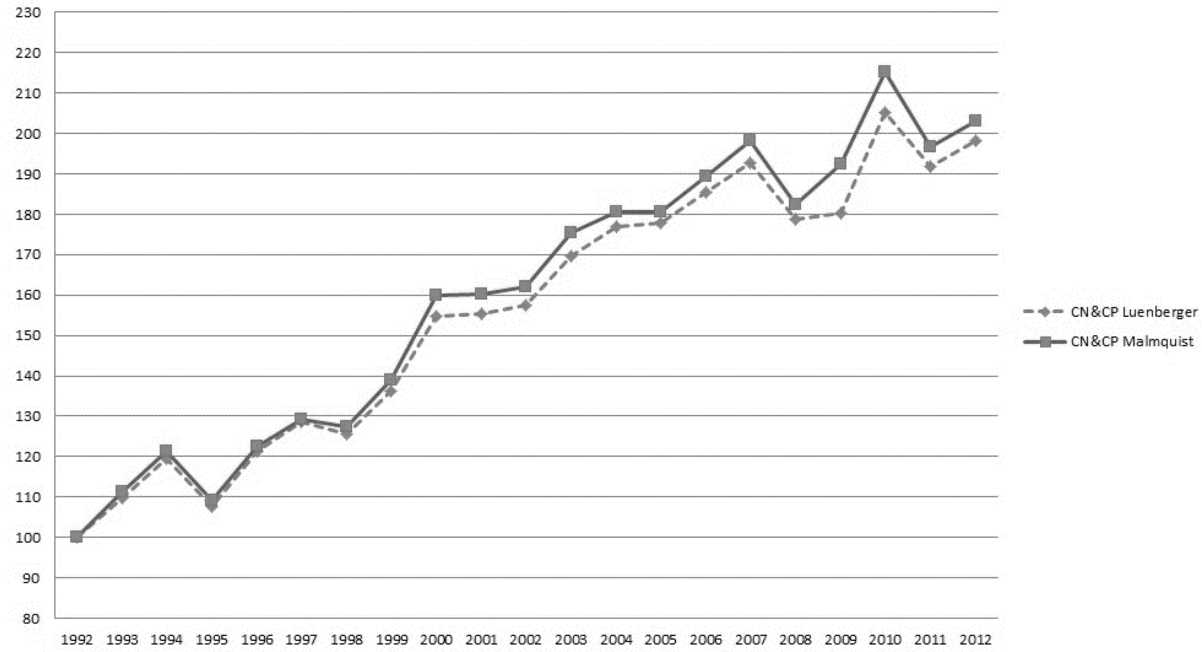
Materials
The Agency approves annually material price indices calculated separately for CN and CP, using the purchase order database for each railway containing the quantity and price for all material purchase transactions during the year. The Agency-approved MPI is reset to 1992=1 and used as the price of materials in this study.

Services
The GDP deflator is used as the price index for the services purchased by the railways.

Markup Factors
The markup factors by which marginal product exceeds factor cost, m_i in the productivity equations, are calculated from data on average revenue per RTM for each of the 14 commodity groups, representing marginal product, and long-run marginal cost per RTM for each of the 14 commodity groups, representing factor cost. Revenue per RTM is calculated from the freight traffic database submitted annually to the Agency by CN and CP, and long-run marginal costs are calculated using the Agency Regulatory Costing Model (ARCM).

RESULTS

Comparative results:



We can see that both models with exact determinations show productivity results that are quiet close, unlike the findings by Boussemart et al (2003) who use a linear approximation.

However, the growth rates shown by the two models are a little different over the period, bringing into question which of the two is more accurate. The Luenberger model, which, theoretically, gives results that depend on the vectors (g_x) and (g_y) defining the nature of input and output movements from one period to another, better seems to reflect the underlying industry reality.

However it is worthy to test how stable is the model when these parameters are modified. The table below shows the Luenberger productivity index for different scenarios on (g_x) and (g_y).

Luenberger Stability:We see that the model is remarkably stable since the index is almost the same for very different configurations of directional vectors.

g_x	1	0	1	0	x	x	0.1 x	10x
g_y	1	1	0	y	y	0	0.1 y	10y
1992	100	100	100	100	100	100	100	100
1993	109.81	109.82	109.81	109.84	109.81	109.78	109.81	109.81
1994	119.54	119.54	119.54	119.57	119.53	119.49	119.53	119.53
1995	107.81	107.78	107.82	107.75	107.71	107.67	107.71	107.71
1996	121.13	121.19	121.13	121.23	121.25	121.27	121.25	121.25
1997	128.43	128.49	128.42	128.52	128.54	128.56	128.54	128.54
1998	125.55	125.51	125.55	125.54	125.51	125.49	125.51	125.51
1999	136.18	136.24	136.17	136.23	136.23	136.24	136.23	136.23
2000	154.78	154.82	154.77	154.82	154.82	154.84	154.82	154.82
2001	155.31	155.36	155.30	155.35	155.35	155.37	155.35	155.35
2002	157.47	157.53	157.46	157.51	157.52	157.54	157.52	157.52
2003	169.34	169.64	169.32	169.58	169.56	169.56	169.56	169.56
2004	176.81	177.13	176.79	177.05	177.03	177.03	177.03	177.03
2005	177.76	178.07	177.74	177.99	177.98	177.98	177.98	177.98
2006	185.23	185.53	185.21	185.48	185.45	185.45	185.45	185.45
2007	192.62	192.94	192.60	192.87	192.85	192.84	192.85	192.85
2008	178.50	178.82	178.48	178.75	178.72	178.70	178.72	178.72
2009	179.73	180.28	179.70	180.39	180.18	180.02	180.18	180.18
2010	204.75	205.45	204.71	205.16	205.27	205.36	205.27	205.27
2011	191.20	191.86	191.16	191.58	191.69	191.77	191.69	191.69
2012	197.79	198.34	197.75	198.05	198.20	198.32	198.20	198.20

Conclusion:

Luenberger and Malmquist models gave very close results and that was quite different than what Boussemart et al (2003) found when working on linear approximations rather than structural forms as we did. We have also found that the Luenberger model gives very stable results when changing the direction vectors (g_x) and (g_y), which means assuming that inputs retract while outputs expand do not make a big difference at the very end. We have therefore showed that these results are quite robust since different mathematical tools serving the same purpose lead to similar results.

In this context, making a choice between use of either Malmquist or Luenberger is not simple, because the two models produce such close results. If it is absolutely necessary to choose one of them, then, as we see input contraction and output expansion over time, we could argue that the Luenberger model is more reflective of the Canadian railway industry. However, the Malmquist model is well reconciled with classical method (growth accounting) and in the meantime it allows the calculation of returns to scale, and provides the opportunity to do further verification. Since Malmquist model gives very close results we would argue that it is good idea to calculate the productivity growth with both models for verification purposes and also in order to have further confidence in the results.

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