MACROSCOPIC FUNDAMENTAL DIAGRAM BASED PERIMETER CONTROL CONSIDERING DYNAMIC USER EQUILIBRIUM[1]

Qiangqiang Guo, Xuegang (Jeff) Ban, Steven Parks

Introduction

Urban congestion and related issues pose a major challenge in many cities around the world. One way to help address these concerns is to control urban traffic more effectively on the regional level so that total system travel time or other measures (such as fuel consumption or emissions) can be minimized. The macroscopic fundamental diagram (MFD) has been recently receiving increasing attention due to its potential to describe high-level regional traffic dynamics. Current studies on MFD-based macroscopic network traffic control can be categorized as: (i) MFD-based perimeter control; and (ii) MFD-based routes guidance. However, existing studies of MFD-based perimeter control (e.g., Geroliminis et al., 2013) do not consider travelers' choice behaviors. Existing works on MFD-based routes guidance (e.g., Yildirimoglu et al., 2015), however, usually assume directly that travelers follow the guidance and neglected traffic control. In practice, traffic control and route choices are coupled. On one hand, traffic control systems try to improve transportation efficiency based on the traffic conditions such as origin-destination (OD) flow distributions and congestion, which depend on travelers' route choices. On the other hand, traffic control strategies that often determine travel times or traffic states impact travelers' route choice. Therefore, considering travelers' route choice behavior when controlling traffic at the macroscopic level is imperative.

This paper aims to fill this gap by combining the two (i.e., network perimeter control and travelers' route choice behavior) simultaneously at the macroscopic level in a rigorous mathematical framework. We first introduce the methodologies including the MFD dynamics, the queue dynamics at the region boundaries, the IDUE principle, and the proposed control method. We also show how to integrate these methodologies as a differential complementarity system (DCS) problem. Then we illustrate that the proposed control method can improve traffic performance through numerical experiments conducted under different demand scenarios.

Methodology

We aim to develop a perimeter control model that considers travelers' route choice behavior. The basic idea is demonstrated in Figure 1. We assume that the whole network is divided into homogeneous regions, each with a well-defined MFD. We model travelers' route choice behavior in this paper by the instantaneous dynamic user equilibrium (IDUE).Travelers choose their routes to minimize their instantaneous travel times, which consist of both the in-region travel times to their destinations and the waiting times at the boundaries they must cross. The dynamics at the boundaries are described by a point queue model. The proposed efficient-range control algorithm controls the flow rate at the boundaries.

Note that Figure 1 is only used to illustrate our model. We next provide details of the major components of the proposed model, including the point queue model, the MDF dynamics, the efficient-range control method, and the IDUE conditions. (Limited by the paper length, we omit the detailed mathematical formulations of these models. Readers who are interested in the model details should refer to the references or contact the authors.):

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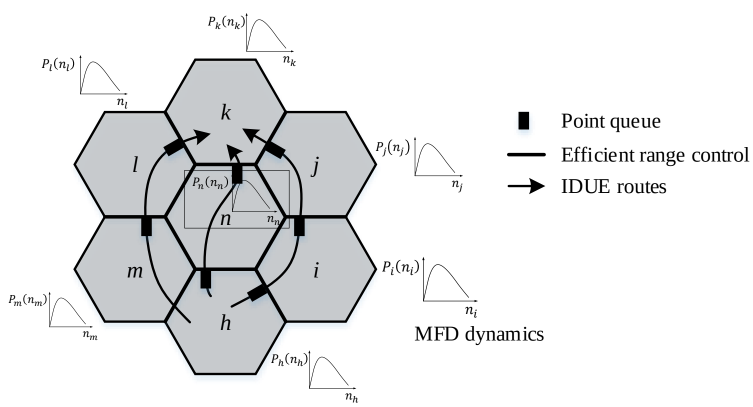
- The point-queue model used in this paper describes the dynamics of the vehicle queuing process at the boundaries between two adjacent regions (Daganzo, 1995). As a concise traffic dynamic model, it can determine the queue length and exit flow for given inflow profiles.
- MFD provides a unimodal, low-scatter, and demand-insensitive relationship between the regional vehicle density and space mean flow in a homogeneous region. The MFD function can be approximated by a third-order function of the accumulation of vehicles in each region (Geroliminis & Daganzo, 2008).
- The IDUE principle assumes that travelers choose the routes that minimize their own instantaneous travel time based on the current prevailing traffic conditions instead of actual (future) conditions (Ran & Boyce, 1996). This principle can be formulated using complementarity conditions (Ban et al., 2012). Besides route choice behavior, IDUE should also consider the flow conservation condition, which can be expressed with complementarity conditions.
- The MFD functions demonstrate that as more vehicles accumulate, the region in question becomes congested and less efficient. The key purpose of our control method is thus to limit the vehicle accumulation in each region to be in the more efficient area by setting a threshold. If the vehicle accumulation exceeds this threshold, the perimeter control decreases the region-inflow rate allowed from the region's adjacent boundaries. Otherwise, the allowed region-inflow rate increases. This control method can be mathematically formulated as an "if-else" condition and further expressed by solving a KKT condition problem.

Figure 1Model overview

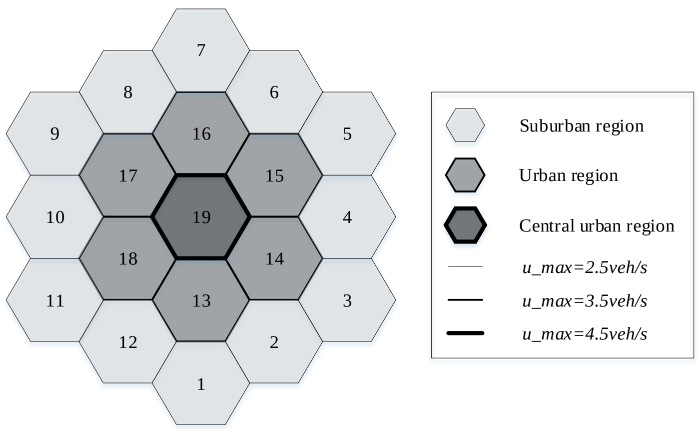


Finally, we integrate the MFD-based perimeter control problem as a DCS problem while considering IDUE. The DCS is a differential-algebraic system consisting of an ordinary differential equation (ODE) parameterized by an algebraic variable that must be a solution toa finite-dimensional state-dependent complementarity problem (Pang &Stewart, 2008; Ban et al., 2012). Generally, the DCS consists of three components: dynamics, complementarity constraints, and initial conditions. In our problem, the MFD dynamics and queue dynamics at the boundaries constitute the dynamics of the DCS. Travelers' route choices, which follow the IDUE principle, determine these dynamics. Thus, the complementary IDUE conditions are part of the constraints. The efficient-range conditions, which serve as another complementary constraint, determine the controlled region-inflow rate. Together, we can formulate the perimeter control problem as a DCS framework. No time delays exist when calculating the point queue dynamics because the inflows to the queues are determined by the MFD functions and IDUE constraints at any instant. Therefore, we can discretize the continuous time DCS problem and solve it using the time-stepping method (Han et al., 2009). The solution's existence and convergence can be established accordingly.

Numerical Experiment

As shown in Figure 2, the studied traffic network consists of three types of regions. The central urban region (i.e., region 19) represents the central urban area, i.e., a city's central business district (CBD). The regions surrounding the central urban region (i.e., regions 13-18) are the normal urban regions. The suburban regions (i.e., regions 1-12) represent the city's suburban areas. Because more travelers use roads closer to the central urban region, we assume maximum inflow ratesof4.5veh/s, 3.5veh/s, and 2.5veh/s for the central urban region, urban regions, and suburban regions, respectively.

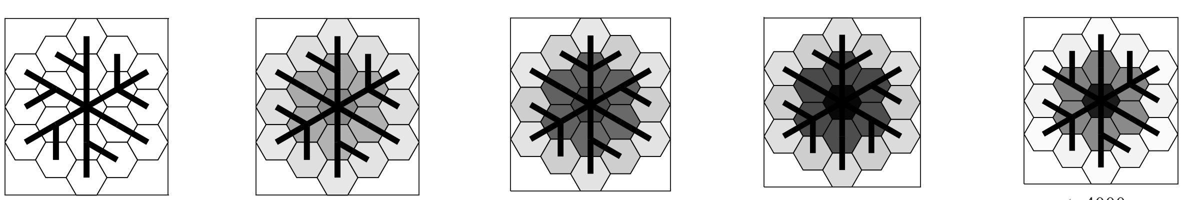
Figure 2 The studied network



We assume that each region has a well-defined MFD function that is modified from the MFD function derived from the data of Yokohama, Japan by Geroliminis and Daganzo (2008) by varying the average trip lengths. A discussion of OD demands between regions is omitted here for brevity. Generally, we use higher OD demands for adjacent regions and lower OD demands for regions far from each other. We load the OD demands for the first 45 minutes to mimic the demand pattern in the morning peak; this is followed by 75 minutes of zero demand to fully clear the network.We illustrate the performance of the efficient-range control by using 4000veh as the threshold.

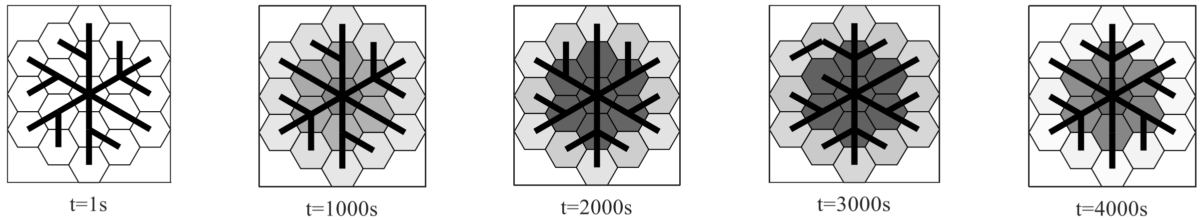
Figure 3 shows the heat map of vehicle accumulations (darker colors show more accumulation) and the routes vehicles take(shown as dark solid lines) from every region to the central urban region (i.e., region 19) at different time stamps. Here a route is defined as a sequence of regions a traveler must traverse from his/her origin region to his/her destination. It can be seen that overall vehicle accumulations with control (shown in Figure 3(b)) are less than those without control (shown in Figure 3(a)) at the same timestamps, indicating reduced overall congestion in the entire network when the control method is applied. The route choices are also different under the controlled and un-controlled conditions. Finally, a comparison of the two figures at t=4000s in Figures 3(a) and 3(b) shows that the network is cleared more quickly when control is applied.

Figure 3The heatmap of vehicle accumulations in each region

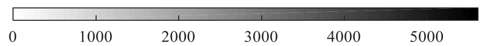


t=1s t=1000s t=2000s t=3000s t=4000s

(a) Without control



(b) With control



Considering that the total demands for both controlled and un-controlled condition are the same given a specific demand scenario, we use the total system travel time (i.e., the summation of the travel times of all travelers in the network) as the performance index to quantitatively evaluate traffic performance. We tested three demand scenarios: light, moderate, and heavy. The results show that the proposed perimeter control method could improve the traffic network performance up to 10.31%under the heavy demand scenario.

Conclusions

This study presented a perimeter control method based on the macroscopic fundamental diagram that implements instantaneous dynamic user equilibrium. We integrated MFD dynamics, the point queue model at the regional boundaries, the IDUE principles, and the proposed control method into a DCS framework. A time-stepping method was used to discretize and solve the continuous DCS problem. We tested the proposed control method under light, moderate, and heavy demand levels. The results showed that the proposed method could improve traffic efficiency up to 10.31% under the specified heavy demand scenario. In future research, we may integrate microscopic traffic control within each region with the proposed perimeter control method because the traffic is managed eventually by real-world facilities such as traffic signals at intersections. In addition, MFD exists only when the congestions are homogenously distributed, which might not hold in practice. Lower-level control could help maintain the region in homogenous conditions to a certain extent. Therefore, integrating the proposed macroscopic-level control with microscopic-level control is imperative to making hierarchical traffic control complete and practical.

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